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RESEARCH AND DEVELOPMENT OF MATERIEL

ENGINEERING DESIGN HANDBOOK

BALLISTIC MISSILE SERIES STRUCTURES

U.S. ARMY MATERIEL COMMAND

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HEADQUARTERS, U. S. ARMY MATERIEL COMMAND

SEPTEMBER 1963

81

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(AMCRD)

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PREFACE

This handbook has been prepared as one of a series on the design of ballistic missiles. It presents selected data on the properties of materials of construction including the effects of elevated temperatures and other operational conditions on ballistic missile structures. Particular attention has been given to the stress analysis of thin shells. In addition the special design considerations applicable to ballistic missile structures are discussed.

The handbook was prepared for the Office of Ordnance Research, Ordnance Corps, U. S. Army. The text and illustrations were prepared by Vitro Laboratories under contract with Duke University, with the technical assistance of Army Ballistic Missile Agency and the Special Projects Branch of Navy Bureau of Ordnance.

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Chapter 1

INTRODUCTION *

1-1. PURPOSE AND SCOPE

The information presented in this handbook has been selected to convey an appreciation of the problems which face the missile structural design specialist and to provide a basis for understanding the considerations which lead to the final structural configuration. Since a complete outline of the theory and practice of structural design and analysis cannot be included in a single volume, the more elementary considerations have been omitted. It is assumed that the reader has a technical background and a reasonable knowledge of the fundamentals of the strength of materials and structural principles. However, the book has been designed for the use of graduate engineers with little or no direct experience in this field rather than for the experienced structural design engineer or structural analyst.

Some of the problems facing the ballistic missile structural design engineer are outlined in this introductory chapter. In Chapter 2, Properties of Materials, elementary and basic considerations in the behavior of materials are introduced, mechanical, creep and thermal properties are discussed, and data on physical properties are presented. Chapter 3, Stress Analysis, gives formulas of stress and strain for basic structural elements under selected loads and boundary conditions. Preliminary stress calculations can be made by the use of the information presented in these two chapters. In the final chapter, Design Considerations, some of the problems facing the ballistic missile design engineer are presented and various possible solutions are discussed.

The principal notation used in this volume is shown in Table 1-1.

* This volume was written by S. V. Nardo and N. J. Hoff, Polytechnic Institute of Brooklyn, Brooklyn, N. Y., and edited by C. D. Fitz, Vitro Laboratories, West Orange, N. J.

TABLE 1-1. PRINCIPAL NOTATIONS

a	Length of flat plate
A	Cross-sectional area
	Radiating area, surface area and area normal to heat flow in radiation, convection and conduction modes of heat transfer, respectively
b	Width of flat plate
	Distance to extreme fibers from neutral plane in a plate
B	Brinell hardness
	Constant, creep laws
Btu	British thermal unit
c	Thermal capacity
C	Distance from neutral axis to extreme fiber
	Constant, creep laws
°C	Temperature, Centigrade scale
D	Diameter
E	Internal energy
	Young's Modulus
E_t	Tangent modulus
E_s	Secant modulus
E_{eff}	Effective modulus
F	Temperature, Fahrenheit scale
F_{cy}	Compressive yield stress
G	Shear modulus
GC	Torsional rigidity
h	Heat transfer coefficient
	Beam depth
	Shell or plate thickness
I	Moment of inertia
k	Thermal conductivity
$k_1, k_2 \ldots$	Constants, creep laws
K	Constant
L	Length
m	Constant, creep laws
M	Bending moment
n	Constant, creep laws
N	Number of stress cycles

TABLE 1-1. (Continued)

p	Constant, creep laws
	Pressure
	Coefficient of first harmonic of thermal stress distribution
p_{cr}	Critical pressure
P	End load on column
q	Rate of cooling
	Rate of heat generation in elemental volume
	Rate of heat flow
	Constant, creep laws
Q	Quantity of heat
r	Multiplying factor
R	Rockwell hardness
	Radius
R_c, R_p, R_t	Stress ratios, interaction formulas
°R	Temperature, Rankine scale
R.T.	Room temperature
t	Thickness
	Time
t_{cr}	Critical time
T	Temperature
u, v, w	Displacements in x, y, z directions respectively
V	Volume
	Shear force
w	Specific weight
x, y, z	Space variables
y	Distance from neutral axis in a beam cross-section
	GREEK LETTERS
α	Thermal diffusivity
	Coefficient of thermal expansion
ϵ	Normal strain
	Emissivity
γ	Half apex angle of cone
λ	Constant, creep laws
ν	Poisson's Ratio
Φ	Rate of radiant energy emission or absorption
ρ	Radius of gyration
σ	Normal stress
	Stefan-Boltzmann constant
σ_{cr}	Critical normal stress
τ	Shear stress
τ_{cr}	Critical shear stress

1-2. BALLISTIC MISSILE STRUCTURAL CONSIDERATIONS

The structural designer of a missile or an airplane has much less freedom in selecting the external shape of his design than the designer of a bridge or building structure. The external form is largely prescribed for him by other requirements. As a consequence his task is a more difficult one. The designer has to work within the space available and must ensure that sufficient strength is provided with the minimum amount of weight. The conservation of weight is of particular importance in ballistic missiles due to the magnitude of the *growth factor*. This factor is the additional weight of the fuel and structural material required by the addition of one pound of payload to the original payload.

The most important difference between the structure of a ballistic missile and most other structures is that the re-entry portion of the missile structure must be designed to withstand high rates of heating. This aerodynamic heating raises the temperature of the structure and influences radically its load-carrying capacity. Of course, aerodynamic heating is present in all supersonic aircraft, but it is in the ballistic missile that it attains its maximum importance.

For this reason a new parameter, namely temperature, enters the analysis of the structure. The elevated temperature, in turn, introduces a second parameter, time, which must also be considered carefully in the analysis of missile structures. While the structure of a conventional ground-based subsonic airplane in principle lasts for an almost indefinite period, the hot structural elements of missiles change their shape continuously under load until they become so deformed that they are incapable of performing their structural duties.

Every structure that has to perform at very high temperature has, therefore, a definite lifetime beyond which it cannot be utilized. The consideration of this lifetime makes the task of designing and analyzing a missile structure much more complicated than similar tasks performed in connection with more conventional structures. The "one-

shot" nature of the ballistic missile introduces a design philosophy which cannot be tolerated in other structures.

1-3. ELEVATED TEMPERATURE EFFECTS

1-3.1. Effects on Material Properties. The most immediate effect of elevated temperature is the deterioration of the load-carrying capacity of the material. Many tests have been carried out to obtain detailed information on this effect. The collected data indicates the deleterious effect elevated temperatures have on a material's physical properties which affect the rigidity, deformation, ultimate strength, fatigue and buckling of structural members. The data also shows that for every structural material there exists a maximum temperature beyond which the load-carrying capacity is reduced to such an extent that the material cannot be advantageously utilized.

1-3.2. Thermal Stresses and Thermal Buckling. The second important effect of aerodynamic heating is the development of internal stresses in the structure. Even in the case of uniform temperature rise in a structural element, external geometrical constraints or a combination of different materials will induce internal stresses. These stresses are sometimes called *temperature stresses.* In almost every practical problem, however, uniform temperatures in a heated structural element are rather unlikely to occur. Two principal reasons are that the aerodynamic heating itself is non-uniform over the exterior surface of the structure, and that the heat capacity of the structure varies from point to point. In consequence of the non-uniform temperature distribution, the material would expand non-uniformly were this possible geometrically. As the natural deformations of the material are incompatible, internal stresses are set up in the non-uniformly heated structural elements to re-establish the continuity of the deformations. These stresses are known properly as *thermal stresses.* However, in this section the term *thermal stresses* shall be used for both the temperature stresses of uniformly heated structures and the thermal stresses of non-uniformly heated structures.

Thermal stresses are of the nature of the secondary stresses of conventional structural engineering. They are a consequence of the deformations and not of the externally applied loads. It is unlikely they will cause failure except, perhaps, in fatigue. However, they may cause unduly large plastic deformations and, in particular, compressive thermal stresses might cause buckling.

Thermal buckling may lead to the development of bulges on the surface of the body or control surfaces of the missile which could interfere seriously with the aerodynamic performance. In particular, the accuracy of the flight path may be endangered. For these reasons the missile structural designer and analyst must be conversant with the calculation of thermal stresses and with the formulas predicting thermal buckling.

1-3.3. Creep and Creep Buckling. A third important effect of elevated temperatures, comparatively little known in conventional structures and machinery, is *creep.* This phenomenon can be observed, for instance, when a heated rod is subjected to a constant tensile load. Under this load the rod elongates continuously with time. Although lead, for example, exhibits creep at room temperature, with commonly employed structural materials the phenomenon is practically non-existent at room temperature. Creep becomes important, however, and may even lead to the elimination of a particular material from use in structures, when the temperature is raised sufficiently.

The first structural effect of creep is the deformation created. It is important that the missile structural designer and analyst be able to predict the time when the deformations will become so large that they substantially change the aerodynamic shape of the missile or that they interfere with the proper functioning of the moveable parts of the system.

A second effect of creep is the occurrence of two entirely new types of structural instability. The first is instability in tension. When

a straight bar is attached to a rigid structure at its upper end and is loaded by a weight at its lower end, the bar elongates continuously with time. As creep deformations of metals take place substantially without any change in the total volume of the material, a one percent increase in the length of the bar must be accompanied by a one percent decrease in its cross-sectional area. However, a constant load applied to a smaller cross-sectional area causes an increased stress in the bar which in turn increases the rate at which creep deformations take place. The creep strain rate is generally a non-linear function of the stress, increasing much more rapidly than the stress. The consequence is a very rapid acceleration of the creep process which can end only in a reduction of the cross-sectional area to the value at which the material will rupture instantaneously under the applied load. The time at which this rupture takes place is known as the *critical time*. This characteristic exists for all structural materials at elevated temperatures and must be known to the structural designer.

A second instability of importance is known as *creep buckling*. This type of buckling differs in many respects from elastic and inelastic buckling as these phenomena are known to the structural engineer. The main difference is that in creep buckling one cannot utilize the concept of a *critical load*. Buckling takes place under any compressive load however small if one waits long enough after the application of the load. On the other hand, just as in the case of tension, a new concept must be introduced, namely that of a *critical time*. An explanation of this phenomenon can be given without difficulty.

Every practical column, as distinguished from the idealized ones underlying some theoretical calculations, is imperfect. This means that the column's axis is never completely straight and the load is never applied to it completely centrally. Consequently the column is subjected to *bending moments*. A bending moment can be calculated as the applied load multiplied by its lever arm, or the distance of the centroid of the cross-section from the line of load application. Under the action of these bending moments the column deforms in creep bending. The increasing curvatures in turn lead to increasing lever arms which again result in increasing bending moments. This is a vicious circle which can end only in the development of very large curvatures and complete collapse of the column. When the behavior of the material is such that the creep rates increase more rapidly than the bending moments, the time necessary for creep buckling to occur is always finite (not infinitely long). Consequently we can properly talk of a critical time. Depending upon the material, the load, and the temperature, this critical time may be a few years, a few months, a few days, a few minutes or even a few seconds. It is therefore most important that the engineer be conversant with the facts of creep buckling when designing missile structures.

1-3.4. Melting and Ablation. Other phenomena which must be treated, if a complete picture of the problems of ballistic missile structures is to be given, are even farther removed from conventional structural analysis. The heating rates experienced by ballistic missiles on re-entry are so large that the surface of the material may melt, sublime or even burn. The designer of the missile must therefore take the necessary precautions in this regard. Some of these precautions are of an entirely aerodynamic nature; if the contours of the missile are properly shaped, the heating that the missile will experience can be minimized. A second precautionary measure is more in the realm of the structural designer; it consists of provisions for thermal protection through shielding the structural material with insulation, through cooling the structure by means of the evaporation of a coolant, or through other means.

In some designs the external metal covering of the missile is used as a *heat sink*. When this is the case, information is required about the thermal conductivity, the thermal diffusivity, and the specific heat of the material. The former quantities determine how rapidly heat will be transmitted from the surface of the missile to its interior, while the last item

indicates how much heat can be stored in the material.

Re-entry has also been accomplished successfully through letting the surface of the missile melt and the molten metal be carried away by the windstream. If sufficient material is available in the structure, the integrity of the missile may not be impaired by this melting. However, the rates of melting and ablation must be calculated, and the designer must also have information about the stability of the process; in particular, whether ridges or pits will be formed or whether the material will be consumed uniformly over the entire surface. Similarly it has been suggested that a missile might be allowed to burn during re-entry just as in the case of a meteor. Again, a great deal of information is needed on this problem before the designer can employ burning as a means of maintaining the integrity of the interior of the structure.

Chapter 2

PROPERTIES OF MATERIALS

2-1. MECHANICAL BEHAVIOR

In the analysis and design of any structure, the physical properties of available materials are of primary importance. With relatively few exceptions, structural engineers in the past were concerned only with those properties which might be classed as purely mechanical. Certain elastic constants, yield and breaking strengths, and fatigue characteristics might be cited as examples. The relatively recent developments in jet and rocket propulsive systems, high-performance aircraft, and missiles have greatly increased the complexity of structural analysis and design problems. Designers of today's high-speed aircraft and missiles are interested not only in the material properties already mentioned, but also in the thermal, creep and plastic characteristics. Indeed, it is certain that this interest will extend to those physical characteristics which were heretofore in the domain of solid-state physics.

A major reason for this extension of interest in material properties is that ballistic missiles are subject to high rates of heating. This heating occurs upon re-entry into the atmosphere and (under certain trajectory conditions) during the ascending phase of the flight. Unless the heat is dissipated the temperature will rise. The load carrying capacity of the structure is affected by the temperatures attained, and also by the length of time at an elevated temperature.

As a general rule, it may be said that elevated temperatures have a deleterious effect on the mechanical properties of materials. For example, the modulus of elasticity, yield stress, and the ultimate stress usually decrease with increasing temperature. It will also be seen that the rupture stress in fatigue, or the number of cycles to failure generally decreases with an increase in temperature.

Data on the properties of some of the more pertinent engineering materials used in missile design are collected and presented in tabular and graphical form at the end of this chapter.

2-1.1. Stress-Strain Curves. Probably the mechanical data most familiar to engineers are stress-strain curves. This information is obtained experimentally by testing a prescribed standard specimen of the material in tension or compression and simultaneously recording the load and elongation. In the simplest of these tests, a cylindrical specimen is loaded in tension in a testing machine, the observed load is divided by the original cross-sectional area to obtain the average stress, and the elongation is measured over a unit length to obtain the strain. When the data are presented graphically in the form of stress as a function of strain, the result is a stress-strain curve. Typical curves are shown in Figure 2-1 for steel and aluminum alloys.

Before the information derivable from these curves is discussed, a brief review of the concepts of stress and strain will be presented. *Stress* is the intensity of the force per unit area acting on a plane. This property is a vector quantity; the normal component of the vector is the *normal* or *direct stress*, and the component in the plane, the *shear stress*. In a cylindrical test specimen, the quantity obtained by dividing the total force by the cross-sectional area is the normal stress acting on a plane perpendicular to the axis of the cylinder With a material that can be considered homogeneous and isotropic, and with a centrally applied load, this definition is reasonably consistent and accurate. Inasmuch as this normal stress is the only stress acting in this simplified case, the stress condition is said to be *uni-axial.*

The stress picture is accompanied by a strain picture with normal and shear strains

developing at each point in the material. Imagine that parallel planes are passed through two neighboring points. The *normal strain* is the relative displacements of these points normal to the planes divided by the original distance between the planes. The *shear strain* is defined as the relative displacement of the two points measured parallel to the planes, divided by the original distance between the planes. Normal strains have the effect of distorting a cubical element of the material without altering the orthogonality between the sides of the element, whereas shearing strains are a measure of the change from the original right angles between the sides.

Figure 2-2 illustrates the separate effects of normal and shear strains on the deformation of a square element of the material. In part (a) of the figure, the element is subjected only to normal strains. This type of deformation would occur, for example, if a normal stress in the x direction were the only stress acting on the element. Note that a strain in the y direction accompanies the strain in the x direction due to stress in the x direction. Part (b) of the figure illustrates the manner in which the element would be distorted if it were subjected to a condition of pure shear stresses. In the most general case there would also be u displacements (x-direction) of the end points of the element.

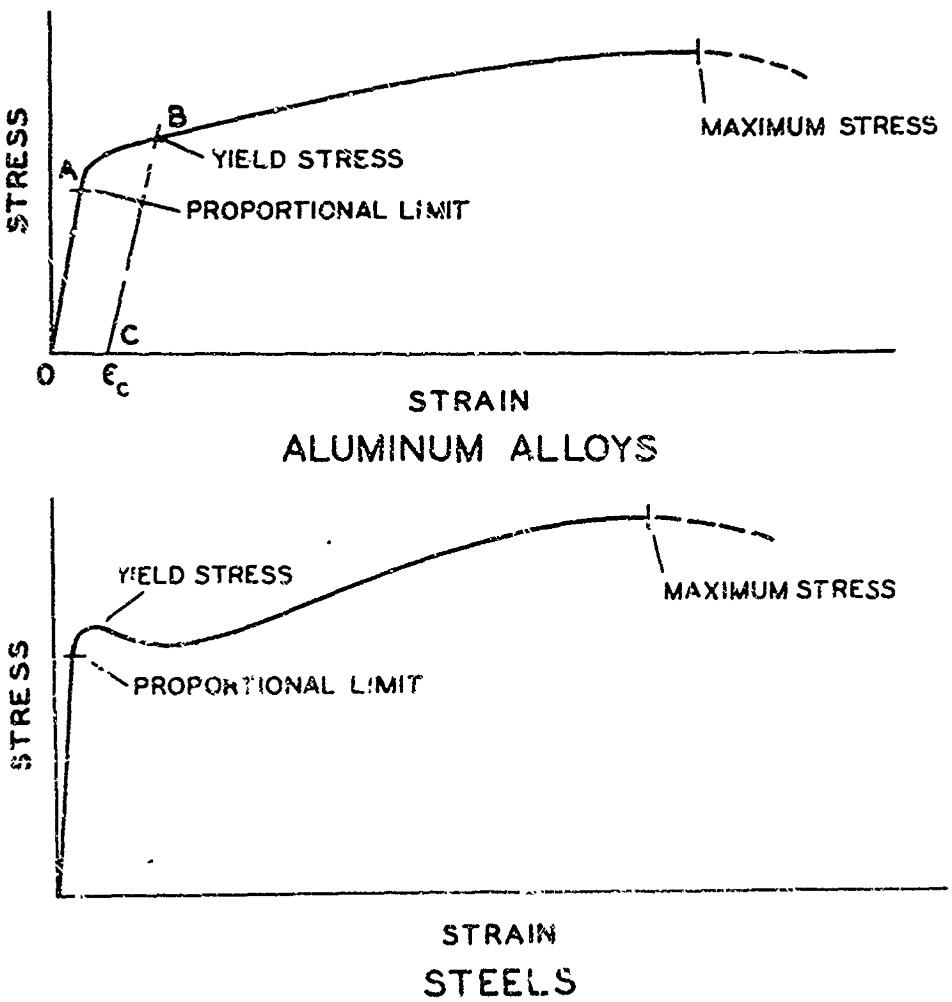


Figure 2-1. Typical Tensile Stress-Strain Diagrams for Steel and Aluminum Alloys

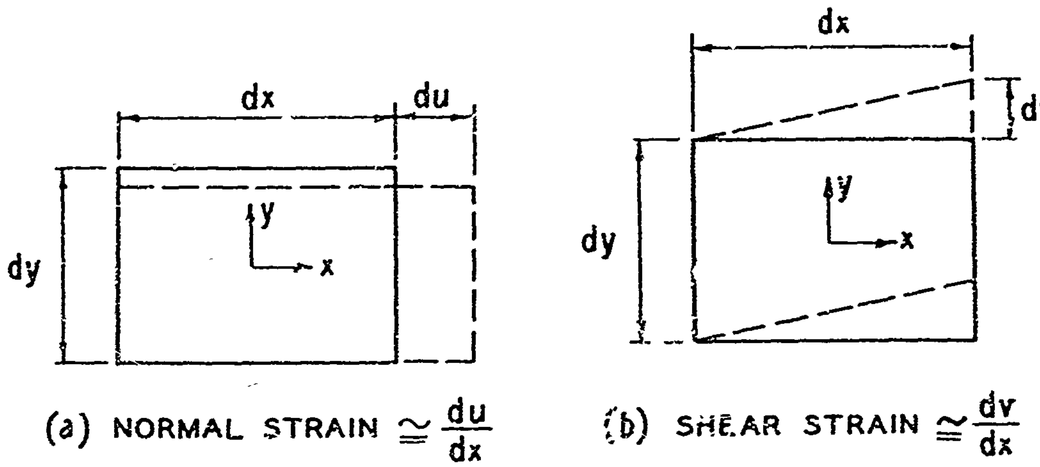


Figure 2-2. Distortion of an Element by Normal and Shear Strains

2-1.2. Proportional Limit. Notwithstanding the relatively simple nature of the test that yields the stress-strain curves of Figure 2-1, some of the most useful and basic properties of the material are deduced from it. It will be noted that both curves exhibit an initial linear relationship between stress and strain. The value of the stress corresponding to the end of this linear relationship is called the *proportional limit.* Provided the stresses remain below the proportional limit, the material remains *elastic,* i.e., it assumes its original shape upon the removal of the load. Further, the material is linearly elastic. It might be well to point out that in reality, a material can be stressed above the proportional limit and still remain elastic. However, for most engineering materials, the proportional and elastic limits are relatively close to each other and are in fact frequently used interchangeably by engineers.

2-1.3. Young's Modulus. The slope of the stress-strain curve, that is the rate of change of the stress with increasing strain in the linear region is called *Young's Modulus.* Physically, it may be considered a spring constant because Young's Modulus is proportional to the force per unit elongation. Although the stress-strain curve gives the value of the spring constant in only one direction, most engineering materials are fairly homogeneous and isotropic, and hence the spring constant or Young's Modulus can be assumed to be independent of both position and direction within the material. Clearly, therefore, this one physical property is of major importance to the structural designer when the design criterion is a rigidity requirement.

2-1.4. Yield. Consider a simple tensile specimen loaded beyond the elastic limit to a stress corresponding to point B in Figure 2-1a. Upon the removal of the load, the stress decreases approximately linearly with strain as illustrated by line BC because only the elastic part of the deformations can be regained. A permanent set is observed when the load is completely removed. The unloading line BC is approximately parallel to the straight line OA whose slope is equal to Young's Modulus. If the material were perfectly elastic, it would have unloaded along the original loading line OAB. For materials whose stress-strain curve has the characteristics shown in part (a) of the figure, the *yield stress* is defined as that value of the stress corresponding to an offset, ϵ_o, of 0.002 For materials whose stress-strain curves are similar to that shown in part (b) of the figure, the first point of horizontal tangency is taken as the *yield point.*

2-1.5. Ultimate Stress. The maximum value of the stress reached in the simple tensile

test is called the *ultimate stress*. Since all stresses are based on the original cross-sectional area of the specimen, the ultimate stress corresponds to the maximum load, and it may be considered as indicative of the strength of the material. Relatively few members in airplane and missile structures, however, are critical in tension. The ultimate stress of a material in compression is not quite as clear-cut because of the column action. Even when the column action is prevented by lateral constraints or by the proportions of the test specimen only some materials such as wood and stone will exhibit a definite value of the ultimate stress. For most metals, an arbitrary criterion must be used to define this quantity. When the design criterion is associated with failure or rupture of the structural member, then the ultimate stress assumes major importance as a material property.

2-1.6. Tangent and Secant Modulus, Modulus of Rigidity. In some structural analyses (column buckling, for example), a so-called *tangent modulus* is used. This quantity is simply the slope of the stress-strain curve at any point. Below the proportional limit, the tangent modulus is everywhere equal to the Young's Modulus. Some investigators use the concept of a *secant modulus*, which is the ratio of stress to strain at any point. Young's Modulus and the secant modulus are also equal below the proportional limit. Torsion stress-strain diagrams can also be plotted from torsion tests on round tubes or round solid sections. The initial slope of this curve, analogous to Young's Modulus, yields the modulus of elasticity in shear, referred to as the *Modulus of Rigidity*.

2-1.7. Poisson's Ratio. For homogeneous, isotropic materials, the Modulus of Rigidity is related to Young's Modulus in the following manner:

$$G = E/2(1 + \nu) \qquad (2\text{-}1)$$

where

E is Young's Modulus
G is Modulus of Rigidity
ν is Poisson's Ratio

Poisson's Ratio is the absolute value of the ratio of the lateral strain to the axial strain in a uniaxially stressed material. As mentioned earlier, a cylindrical tensile test specimen subjected to an axial load produces an axial strain. This axial strain is always accompanied by a lateral strain smaller in magnitude, and opposite in sign [Figure 2-2(a)]. For most engineering materials Poisson's Ratio has a value of approximately 0.3.

2-1.8. Fatigue. The physical properties of materials discussed so far have been based on static tests in which the load is slowly applied without repetition. It is well known that some structural elements are subject to vibrations which cause the stresses to fluctuate. Under these conditions, failure can occur at a stress level which may be considerably lower than the ultimate stress previously discussed. This type of failure is known as *fatigue* failure.

The fatigue strength of a structural element is probably the least amenable to analysis and hence the most difficult of the various design strengths to evaluate. Unfortunately, its determination is not confined to a knowledge of the nature and magnitude of the fluctuating stresses and some simply physical properties. It is known that the fatigue strength of a specimen is affected by size, local discontinuities, and heat treatment, to mention a few of the parameters. Investigators in this field have nevertheless made significant strides by their analytical studies and extensive experimental programs.

The data used by design engineers in determining fatigue strength are based on tests of standard specimens (smooth or notched) subject to controlled fluctuating stresses, in which the number of cycles to failure is recorded. Part (a) of Figure 2-3 shows a periodic variation of stress to which a fatigue specimen may be subjected and defines the important stress parameters affecting fatigue life. In part (b) of the figure, a characteristic curve is presented which gives results of fatigue tests on a specimen. The curve is drawn for a mean stress value of zero, and relates the stress to the number of cycles to

failure. Curves of this type are commonly called S–N diagrams. The value of the greatest stress which the material can withstand for an infinite number of cycles without failure is called the *endurance limit*. This is a simple and admittedly crude measure of the fatigue strength of the material. If the endurance limit so defined cannot be detected with a reasonable amount of testing, a practical endurance limit is often introduced as the stress at which 10^7 load cycles do not cause failure.

For the case of zero mean stress, it will be noted that the maximum stress is equal to the stress amplitude. The maximum and minimum stresses are equal in value, opposite in sign, and correspond to a case of completely reversed loading. When the mean stress is other than zero, the stress amplitude corresponding to fatigue decreases with increasing mean stress. Typical curves at room and elevated temperatures are presented in the literature.[1]*

* Superscript numbers refer to the bibliography at the end of this chapter.

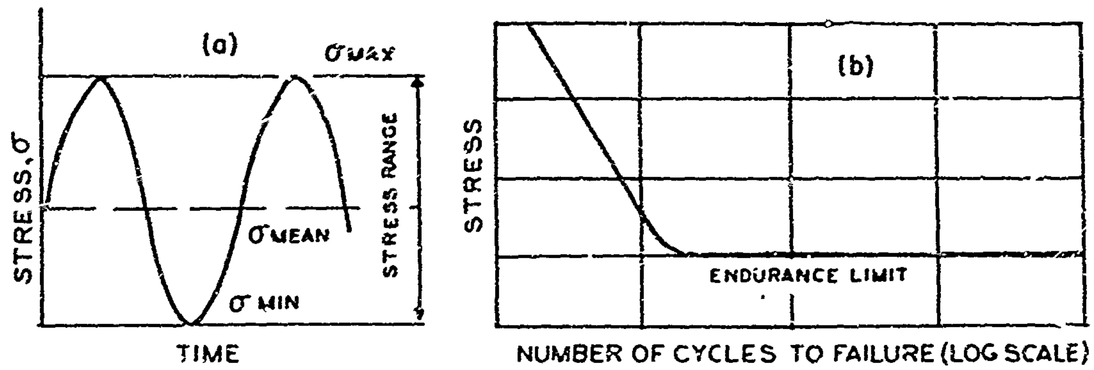


Figure 2-3. Fatigue Loading and Typical Stress-Cycles to Failure (S-N) Curve

2-1.9. Other Mechanical Properties. Of great importance to the missile designer is the plastic behavior of materials. *Plasticity* is related to the ability of a material to sustain a permanent deformation without fracture. A perfectly plastic material, for example, would require no increase in stress to produce a continuing plastic deformation once the yield stress was reached; it would be represented by a segment of the stress axis and a horizontal straight line on the stress-strain diagram. The importance of plasticity can be illustrated by considering a structure which is subjected locally to a high stress level. If the material is ductile, plastic flow occurs, and the load is redistributed more uniformly. Neighboring points in the material absorb the load which is shed at the point of stress concentration, thus lowering the intensity of the maximum stress and lessening the possibility of early failure. A *brittle* material does not readily flow plastically and hence there is the danger of brittle fracture. Quantitative measures of the ductility of a material are the percent elongation in 2 inches of a test specimen and the reduction of area at fracture in a tension test.

The *hardness* of a material is a measure of its ability to resist permanent surface deformation under applied concentrated loads. There are several standard methods of measuring this quantity. Values of hardness for several materials are given in the tables at the end of this chapter. Approximate correlation between the various hardness scales are given in the literature.[x] It can be shown that for many steels the tensile strength is proportional to the hardness.

2-2. CREEP BEHAVIOR

Creep may be defined as the continuing deformation that occurs in a material subjected to a constant stress. At room temperatures, the creep of engineering materials is unimportant, but at elevated temperatures

creep effects may yield important design criteria.

Creep data are generally presented in the form of strain as a function of time as shown in Figure 2-4. Specimens are subjected to a constant load, which corresponds to a constant stress level based on the original cross-sectional area of the specimen. The instantaneous value of the strain, ϵ_a, corresponds to the elastic deformation calculable from the load, original cross-sectional area and Young's Modulus. With the passage of time, the strain increases; the strain rate is high at first and gradually decreases to a constant value. The portion of the creep curve in which the creep rate is changing is called the *primary stage* or *primary creep*. The portion corresponding to a constant rate of strain is called the *secondary stage*, or *secondary creep*. In the *tertiary stage*, the strain rate suddenly increases until the specimen ruptures.

Removal of the load during the secondary stage results in the recovery of the elastic strain ϵ_a and sometimes also of that portion of the creep strain ϵ_b obtained by extrapolating the straight line of the secondary creep stage to t = 0. Some solid state physicists attribute this to a release of internal stresses which occurred during the loading and creep processes. It should be noted that the true physical nature of creep has not yet been established with any degree of finality.

Elongations in the specimen that occur with the passage of time are accompanied by a contraction of the specimen in the lateral direction. This decreases the cross-sectional area and increases the value of the true stress which is the load divided by the true cross-sectional area. This process accelerates rapidly and culminates in rupture of the specimen when the greatly reduced cross-sectional area can no longer support the applied load. The portion of the creep curve corresponding to increasing rates of creep is the tertiary stage. It should be mentioned that metallographic changes in the material (for instance those caused by annealing during the test) can also lead to increasing rates of creep.

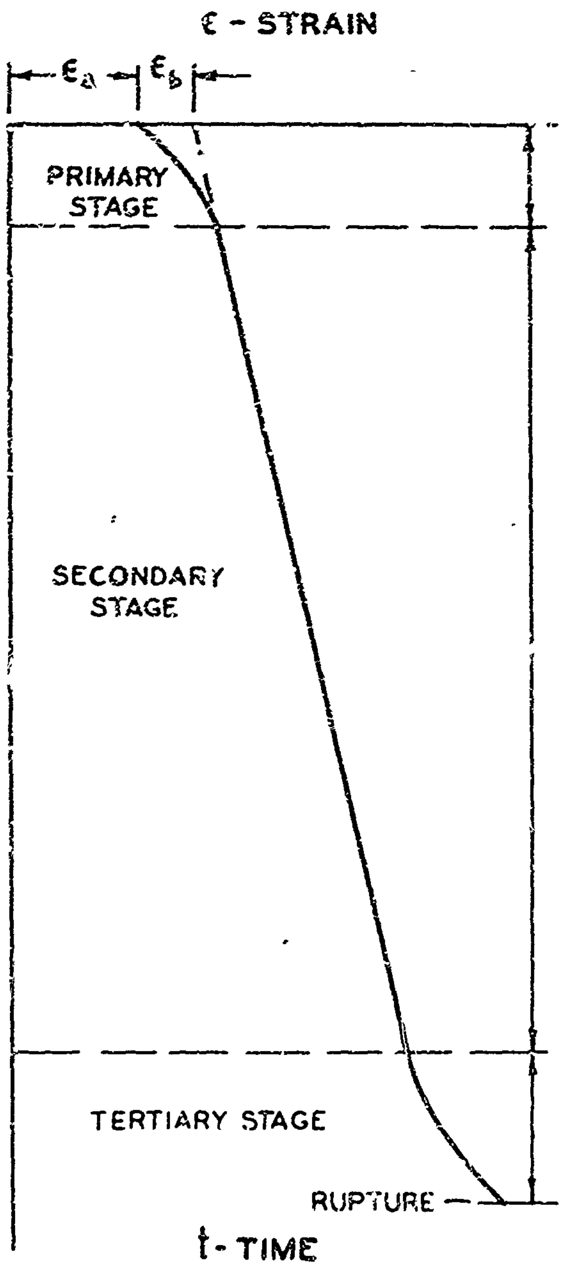


Figure 2-4. Typical Presentation of Creep Data

2-3. HEAT TRANSFER

It is customary to distinguish three basic modes by which heat is transferred from one point to another: (1) *radiation*, (2) *convection*, and (3) *conduction*. While the actual mechanism underlying the transfer of heat energy is not completely understood, the phenomenon is observed to be consistent with certain physical laws postulated by various investigators. The particular law which is applicable can be established by recourse to experimental methods.

Whatever the mode of heat transfer, the missile designer is basically interested in temperature distribution either as a function of space or of space and time. The temperature distribution is required to determine heat transfer rates, to make possible the computation of thermal stresses, to determine the change in material physical properties, or to cope with a problem in melting, ablation or burning.

2-3.1. Radiation. The transfer of heat energy by radiation is probably best exemplified by considering the transfer of heat from the sun to the earth. Thermal energy from a hot body, the sun, is converted into electromagnetic energy which is transmitted through space to a cold body, the earth, where it is reconverted into thermal energy. The radiant energy reaching the cold body can be absorbed, reflected or transmitted through the body. Since most engineering materials are opaque, the transmissivity of these materials is zero, hence the reflectivity and absorptivity only need be considered. The concept of a *black body* is the limiting case of a body with 100% absorptivity and zero reflectivity. For a given absolute temperature, no body emits or absorbs more heat than a black body. The rate at which the radiant energy is emitted or absorbed by a black body is given by the Stefan-Boltzmann law:

$$\Phi_b = \sigma A T^4 \qquad (2\text{-}2)$$

Φ_b = rate of radiant energy emission or absorption by a black body, Btu/hr

σ = Stefan-Boltzmann constant 0.174 × 10^{-8} Btu/(hr)(ft^2)(F^4)

A = radiating area, ft^2

T = absolute temperature, F

A so-called *gray body* will reflect some of the radiant energy received, hence the emissive power of a gray body will depend upon the proportions of radiant energy that are absorbed and reflected. It should be noted that the radiating power of a body, in addition to being proportional to the fourth power of the temperature, is also dependent upon the physical properties of the body and varies with the wave length. It is usual to neglect the dependence of the radiant power on the wave length. The corresponding equation for Φ for a gray body is written

$$\Phi_{gr} = \epsilon\sigma A T^4 \qquad (2\text{-}3)$$

where ϵ is the emissivity of the gray body. The emissivity for some engineering materials is given in the literature.[15]

Two final relations should be considered. The first is that radiant interchange between surfaces is governed by the above principles if there is a non-absorbing medium between the surfaces. Second, the radiant interchange also depends upon the inverse square of the distance between the surfaces and the relative orientation of one surface with respect to another. If it is assumed that the radiation from a heated surface has the same flux in all directions, the orientation or "view" becomes purely geometrical in nature. These factors are also given in the literature.[15]

2-3.2. Convection. The transmission of heat by convection differs materially from the transmission of heat by radiation. In the case of convective heat transfer, heat is conveyed from one point to another by the movement of a fluid stream; i.e., the transport process is associated with the actual movement of a fluid stream. Convective heat transfer may be *natural* or *forced* depending upon whether the fluid motion occurs by virtue of the natural tendency of the heated, lighter element of the fluid to rise, or whether a pump, blower or other means are used to produce the fluid motion. It shall simply be mentioned here that natural convective cooling of a surface area A at the boundary of a solid is proportional to $(T_w - T_a)^{5/4}$ where T_w and T_a are the surface and ambient fluid tempera-

tures respectively. In practice, a linearized approximation can usually be taken and the analytical expression for the convective cooling of a surface can be written as

$$q = hA(T_w - T_r) \qquad (2\text{-}4)$$

where

q = rate of cooling, Btu/hr
h = heat transfer coefficient, Btu/(hr)(ft²)(F)

Strictly speaking, the convective cooling of a surface is a combination of heat transfer both by convection and conduction within the fluid. The reader will note that the above equation is similar to Fourier's basic heat conduction Equation (2-5). The reason for its appearance here lies in a concept which essentially converts the convective cooling problem to a conduction problem. This concept considers that there is a film adjacent to the surface through which there is a linear temperature drop from T_w to T_r. The effective film thickness is difficult to measure and is incorporated in the heat transfer coefficient h which also is a function of the physical and dynamical properties of the media involved.

2-3.3. Conduction. Of greatest interest to the missile structural designer is the transmission of heat by conduction. This process is generally interpreted as a simple molecular interchange of kinetic energy from one molecule to an adjacent molecule by virtue of elastic impacts. The heat flow is observed to pass from the molecules at high temperature to those at a relatively lower temperature.

It has been customary in the analysis of the temperature distribution in a structure to consider convective and radiant heat transfer as the means by which the external air flow heats the structure, and to disregard these modes of heat transfer when the flow of heat in the interior of the structure is sought. Two reasons can be mentioned in defense of this simplified approach; first, the equations of conduction have been solved rigorously for many more cases than those governing the other two modes of heat transfer; and second, the results obtained from an analysis based on conduction alone always correspond to higher thermal stresses than those based on a consideration of all three modes. In a solid structure the only mode of heat transfer is conduction; in hollow structure, however, convection and radiation can significantly modify the temperature distribution calculated on the basis of conduction alone.

It might be mentioned that a relatively complete analysis of simultaneous effects of conduction, convection, and radiation is possible through the use of computers.

The basic law of heat conduction has its origin in the fundamental work of the French mathematician Jean Fourier. Fourier's Law states that the instantaneous rate of heat flow in a one-dimensional body is given by the product of the area perpendicular to the flow, the rate of change of temperature in the direction of flow, and the thermal conductivity.

$$\frac{dQ}{dt} = q = -kA\frac{\partial T}{\partial x} \qquad (2\text{-}5)$$

In the above expression

Q = quantity of heat, Btu
t = time, hr
q = rate of heat flow, Btu/hr
k = *thermal conductivity*, the amount of heat per unit time flowing through a unit area under a unit temperature gradient, Btu-in./ft²hr F
A = area normal to heat flow, ft²
T = temperature, F
x = space coordinate in flow direction, ft

The units used are British Gravitational units, and the minus sign is arbitrarily chosen to make Q a positive quantity. In the form shown, the temperature T is assumed to be a function of space and time (transient conduction), although it may be a function of space alone (steady conduction). Taking the case of steady conduction in one dimension and one for which A is not a function of x, Fourier's Equation is

$$q = -k\frac{A}{L}(T_2 - T_1) \qquad (2\text{-}6)$$

where

L = total path length $(x_2 - x_1)$
T_2 = temperature at x_2
T_1 = temperature at x_1

Consider, for example, a rod of uniform cross-sectional area A and length L. For a constant value of the heat flow q, the temperature difference $(T_2 - T_1)$ is inversely proportional to the thermal conductivity k. Thus a material of high thermal conductivity will allow a given heat flow at a lower temperature difference than a material of lower thermal conductivity. The missile designer interested in conducting the heat input on the surface of a structure to the interior with a minimum temperature rise at the surface, will choose a material having high thermal conductivity (assuming all other material properties are the same).

In the transient state of heat conduction, the quantity of heat entering and leaving a volume element at any instant is not equal. The increase in internal energy, dE is

$$dE = c\,w\,V\,\frac{\partial T}{\partial t}\,dt \qquad (2\text{-}7)$$

where

c = *thermal capacity* of the material, the amount of heat energy the material can absorb per unit weight for a unit temperature rise Btu/lb./°F

w = specific weight of the material, lbs/ft^3

V = volume, ft^3

A missile designer interested in using the structural material as a heat sink, would choose a material of high thermal capacity, again assuming that other material properties were equal. The equation of continuity for the heat flow through a volume element in three dimensions can be written

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + c\,w\,\frac{\partial T}{dt} + q = 0 \qquad (2\text{-}8)$$

where the effect of the heat input q, Btu/ft^3-hr, has been added. In the most general case, k, c, w and q are functions of x, y, z and T; q may also be time dependent. As written, Equation (2-8) is valid for an inhomogeneous, but isotropic solid. Moreover, the equation is non-linear if k, c, or w depend on T. Many cases of practical interest are covered by taking q as a function of T only, and assuming k, c, and w constant. Under these conditions, the most general heat conduction equation reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{q}{k} = 0 \qquad (2\text{-}9)$$

where

$$\alpha = \frac{k}{cw} = \textit{thermal diffusivity}\ \text{ft}^2/\text{hr}$$

For systems that do not contain heat sources or sinks, q = 0, and Equation (2-9) becomes the *Fourier Equation.* If sources and sinks are present but the temperature is steady, $\frac{\partial T}{\partial t} = 0$, and Equation (2-9) reduces to *Poisson's Equation.* Finally, for steady conduction in systems free of sources and sinks, the well-known *Laplace Equation* is obtained. This linear partial-differential equation appears frequently in many branches of science. Solutions are discussed in the literature.

The thermal diffusivity is a derived material property important to the missile designer as a measure of the ability of a material to absorb or diffuse heat. As defined by Equation (2-9) thermal diffusivity is essentially the ratio of the thermal conductivity to the specific heat. The time required to bring a material to a specified temperature level varies inversely with the thermal diffusivity and directly with the square of the conducting path length.

2-3.4. Thermal Expansion. When an engineering material is heated, it expands in all directions in a manner that is dependent upon the material and the temperature rise. For the single dimension case the ratio of the unit elongation per unit temperature rise is called the *coefficient of thermal expansion.*

$$\alpha = \frac{d\epsilon}{dT} \qquad (2\text{-}10)$$

where

α = coefficient of thermal expansion 1/°F

ϵ = elongation per unit length

Engineers find it more convenient to define a mean coefficient of thermal expansion as follows:

$$\alpha_m = \frac{\int_{T_0}^{T} \alpha\, dT}{T - T_0} \qquad (2\text{-}11)$$

where T_o is a reference temperature. The unit elongation can be easily calculated after α_m is computed and plotted as a function of temperature from

$$\epsilon = \alpha_m(T - T_o) \qquad (2\text{-}12)$$

If the unit elongation is required for a temperature rise other than from the reference temperature, say T_1 to T_2, then

$$\epsilon = \alpha_{m2}(T_2 - T_o) - \alpha_{m1}(T_1 - T_o) \qquad (2\text{-}13)$$

Consider the case of a slender rod of original length L_o, heated to a temperature T_1 from a reference temperature value T_o. The increase in the length of the rod due to the effects of thermal expansion is required. Taking $T_1 - T_o = T = 100°F$ as the temperature rise, $L_o = 1$ ft, and assuming L_m for the material has a value of 10^{-5}, the unit elongation from (2-12) is

$$\epsilon = \alpha_m(T_1 - T_o) = 10^{-5}(10^2) = 10^{-3}$$

The elongation of the rod is

$$\Delta L = \epsilon L_o = 10^{-3}\text{ ft} \qquad (2\text{-}14)$$

It might be well to point out the implicit assumption that the rod is not subject to any external constraints. If the rod is constrained to remain at its original length, then the temperature rise would produce normal stresses in the rod corresponding to the stress required to produce an equal and opposite unit elongation. The value of this (thermal) stress is obtained by multiplying the unit elongation by Young's Modulus:

$$\sigma = E\epsilon \qquad (2\text{-}15)$$

Hence if Young's Modulus for the rod used in the above example is 10^7 psi, the stress can easily be calculated:

$$\sigma = 10^7(10^{-3}) = 10{,}000\text{ psi}$$

Partial restraints at the ends of the rod produce thermal stresses which are between the values of zero thermal stress for unrestrained ends and the thermal stress calculated above for fully restrained ends.

Expansion of the rod in the lateral directions may be calculated in a similar manner. The coefficient of thermal expansion is usually assumed to be independent of direction. Since most engineering materials can be considered isotropic, the assumption is fairly good. The quantity α is also called the coefficient of linear expansion. Coefficients of thermal expansion for some engineering materials are given in the tables and graphs at the end of this chapter.

2-4. SELECTED DATA ON PROPERTIES OF MATERIALS OF CONSTRUCTION

Collected in the pages that follow are selected data on material properties pertinent to missile design. Data are included on materials that appear to have promise for application in designs of the future. These data have been gleaned from a number of references which are listed in the bibliography. One of the principal reasons for collecting the information has been to give the reader a quantitative feeling for the order of magnitude of the physical properties involved. In some instances, the variations in these physical properties for different alloys of the same material are presented, but this is not considered of paramount interest. In addition to the objective mentioned, the curves are set up to afford an easy comparison between one material and another. The frequency with which time and temperature appear as parameters may be noted.

Another objective in the presentation of the data is to point up the form of presentation which finds favor with the missile designer and stress analyst. In many instances data of a particular kind are shown only for one alloy of one material. This is particularly true in the graphical presentation of creep data, and stress-strain and tangent modulus curves as functions of temperature.

Tables 2-1 through 2-5 cover several alloys each of steel, aluminum, titanium, magnesium, and a few ceramics. In this compilation the room temperature mechanical properties are given for each alloy and for a selected form and condition. Young's Modulus, yield and ultimate strengths are presented on the basis of tensile test data. The yield stress is based upon a 0.2% permanent offset condition. The elongation is the percent change in length measured over 2 inches of a test specimen.

Table 2-6 summarizes the physical properties of various materials: density, melting

point, specific heat, thermal conductivity and coefficient of thermal expansion. In this compilation, only one or two alloys of steel, magnesium, aluminum and titanium are listed, but the properties of materials such as copper, tungsten, molybdenum, oxides, carbides and borides are presented because of the attractiveness of one or more of their properties at elevated temperatures. Mean values of the thermal properties are given together with the temperature range within which they are applicable. For a number of materials, the thermal properties are plotted as a function of temperature and a reference to the corresponding figure is given.

In Figures 2-5 and 2-6 the modulus of elasticity and the yield stress are plotted as functions of the temperature for a selected number of typical materials. As already mentioned, the principal purpose is not only to show the variation of these properties for a material, but also to indicate the relative magnitudes and the behavior between different materials. It is significant to point out that these properties are affected by the rate of heating, soaking time at test temperature, and the strain rate. Most of the mechanical properties shown are for moderate heating and strain rates, and are soaked for one-half hour or more at the test temperature. Short time test results, which are not quite as plentiful, indicate an increase in the modulus of elasticity and yield stress with an increase in the heating rate and strain rate and with a decrease in the soaking time.

The weight-strength and cost-strength ratios of one alloy each of aluminum, magnesium, titanium, steel and nickel are presented in Figures 2-7 and 2-8. Note how effectively the curves give an approximate idea of the maximum useful temperatures of the various alloys based upon these criteria. Cost data are listed in Table 2-7 and strength data are based on the tensile yield strength given in Figure 2-6. The reader is cautioned against arriving at any broad generalization based upon criteria of this type.

Stress-strain curves at various temperatures are shown for an aluminum alloy and a steel alloy in Figures 2-9 and 2-10 respectively. A word of caution is made in connection with the interpretation of the curves shown in these figures. The stress-strain curves shown are obtained at a constant temperature given by the intersection of the curve with the temperature axis. The stress-strain curves are spread in their correct position on the temperature axis (abscissa) in order that curves at intermediate temperatures may be interpolated. For any given stress-strain curve, the strain at any stress level may be obtained by interpolation between constant strain lines or by using the strain scale (shown only in Figure 2-9) and measuring from the point of zero stress. An illustrative example is given in Figure 2-9 for further clarification.

Tangent modulus data at various stresses are shown for an aluminum alloy in Figure 2-11. The values are shown as ratios of Tangent Modulus, to Young's Modulus, E, at various temperatures. A tabulation of mechanical properties at room and elevated temperatures for an aluminum alloy is given in Table 2-8. Creep and fatigue data are included.

The creep data given in Figures 2-12 through 2-17 are in the forms preferred by structural designers and analysts. In Figure 2-15, the reader should note that the elastic strain is subtracted from the total strain to obtain the curves. The log-log plot of minimum creep rate as a function of stress in Figure 2-16 is particularly useful in the formulation of creep laws for engineering materials. Finally, in the master creep curves of Figure 2-17, the parameters are total strain, and a so-called Larson-Miller parameter which includes both the time and temperature effect. The comments made in connection with Figures 2-9 and 2-10 also hold for this figure except that temperature is replaced by the Larson-Miller parameter.

Inasmuch as fatigue considerations are relatively unimportant in missile type structures, only one set of curves is given for an alloy of aluminum in Figure 2-18.

The final set of figures, 2-19 through 2-21, show the thermal properties of a wide variety of materials as functions of temperature. To enable a comparison between the various materials, each figure shows the variation of a particular thermal property.

TABLE 2-1. MECHANICAL PROPERTIES OF SOME TYPICAL ALLOY STEELS

Alloy	Form	Condition	Young's Modulus, 10^6 psi	Shear Modulus, 10^6 psi	Density, lbs/in³	Tensile Strength, 10^3 psi	Yield Strength, 10^3 psi	Elongation, Percent	Hardness Rc ♠	Hardness Bhn δ
4130 8630	Tube Sheet Plate	t<0.188	29.0	11.0	0.283	95	75			
	Tube	Heat Treated				125	103	23		
8630	Sheet	to Obtain	29.0	11.0	.283	150	132	18.5		
4340	Bar	Tensile Strength Indicated				200	176	13.5		
18-8 Type 301	Sheet Strip	Annealed	29.0	12.5	.280	75	30			
		Cold Rolled ½ Hard	26.0	11.5	.286	150	110	18	32	
		Cold Rolled & Heat Treated ½ Hard	27.0	11.5	.286	150	120	9	41	
18-8 Type 304	Sheet	Annealed	29.0		.286	85	30	60		170
		Cold Rolled				185	160	8		400
Stainless 422M		Heat Treated				150-175	120-140		38	
Stainless 17-7H		TH 1050	27.6			180	150	6		
Inconel X	Rods	Heat Treated & Hot Rolled	30.5	11.5	.300	162	92	24		
		Hot Rolled & Aged	30.5	11.5	.300	184	132	24		340

♠ Rockwell C scale.
δ Brinell hardness number.

TABLE 2-2. MECHANICAL PROPERTIES OF SOME TYPICAL ALUMINUM ALLOYS

Alloy	Form	Condition	Young's Modulus, 10^6 psi	Shear Modulus, 10^6 psi	Density, lbs/in^3	Tensile Strength, 10^3 psi	Yield Strength, 10^3 psi	Elongation, Percent	Brinell Hardness Number
2024-T4	Sheet Plate	Heat Treated t:0.25-0.50	10.5	4.0	0.100	64	40	12	120
2024-T4	Sheet Plate	Clad Heat Treated t:0.25-0.499	10.5		.100	62	40	11	
2024-T4	Sheet Plate Extrusions					68	46	22	
2014-T6	Sheet Plate Extrusions					70	60	13	
2014-T6	Sheet Plate	Heat Treated and Aged t:0.040-0.499	10.4	3.95	.101	68	60	8	135
5052-H34	Sheet	½ Hard t:0.008-0.249	10.1	3.85	.096	34	24	3-7	
7075-T6	Sheet Plate	Heat Treated and Aged t:0.040-0.249	10.3	3.9	.101	77	67	8	
7075-T6	Rolled Bar, Rod, Tubing, Shapes	Heat Treated & Aged t ≤3.00	10.3	3.9	.101	77	68	10	
X2020-T6	Sheet, Forged Rod		11.1			86	79	6	
X2219-T6	Sheet, Plate, Extrusions, Forgings					60	38	13	100
APM-M257	Extrusions Forgings	Aluminum Powder Products	10.6		.099	37	24	17	70
APM-M486	Tubing					48	34	15	
XA 140-F	Casting Alloy	As Cast				34	29	1	95 (400°F)

TABLE 2-3. MECHANICAL PROPERTIES OF SOME TYPICAL ALLOYS OF MAGNESIUM, MOLYBDENUM, AND NIOBIUM

Alloy (ASTM)	Form	Condition	Young's Modulus, 10^6 psi	Shear Modulus, 10^6 psi	Density, lbs/in^3	Tensile Strength, 10^3 psi	Yield Strength, 10^3 psi	Elongation, Percent
			MAGNESIUM					
AZ31A	Sheet	Annealed	6.5	2.4	.0645	32	15	12
AZ31A	Sheet	Hard Rolled	6.5	2.4	.0645	39	29	4
AZ31B	Bars Rods Solid Shapes	As Extruded	6.5	2.4	.0645	33	20	7
AZ80A	Bars Rods Solid Shapes	Extruded & Aged	6.5	2.4	.0653	47	30	4
AZ61A	Round Tubing	As Extruded	6.5	2.4	.0649	36	15	7
AZ92A	Sand Castings	As Cast	6.5	2.4	.0657	20	10	
AZ92A	Sand Castings	Heat Treated & Aged	6.5	2.4	.0657	34	18	
AZ80A	Forging	As Forged	6.5	2.4	.0653	42	26	5
			MOLYBDENUM					
Molybdenum ½% Ti	Sheet		46.0		.369	132	99	31
			NIOBIUM					
Niobium		Hard	15.0	5.4		43–58		2
Niobium		Annealed				21–26		10

TABLE 2-4. MECHANICAL PROPERTIES OF TITANIUM AND SOME OF ITS TYPICAL ALLOYS

Alloy	Form	Condition	Tensile Strength, 10^3 psi	Yield Strength, 10^3 psi	Elongation, Percent	Reduction of Area, Percent
High-Purity Titanium			34		54	
Commercial Titanium	Bar	Annealed	95	80	27	47
Commercial Titanium	Sheet	Annealed	95	78	23	48
Ti-5Al-2.5 Sn	Sheet	Annealed	138	129	15	43
Ti-8Mn	Sheet	Annealed	140	123	14	26
Ti-6Al-4V	Bar	Annealed	136-154	121-145	14-18	26-50

TABLE 2-5. MECHANICAL PROPERTIES OF SOME TYPICAL CERMETS, METALLIC REFRACTORIES, AND CERAMICS

Composition	Young's Modulus, 10^6 psi	Tensile Strength 10^3 psi		Vickers Hardness Number
		Room Temp.	1000°C	
75 TiC, 15 Ni, 5 Co, 5 Cr	60	175	80	3200
64 TiC, 6 NbC, 25 Ni, 5 Mo	55	220	100	1400
20 Cr_3B, 80 Cr-Mo		60-80	110	650
95 NiAl, 5 Ni		130	65	400
70 Al_2O_3, 20 Cr	52	55	33	

TABLE 2-6. PHYSICAL PROPERTIES OF SOME TYPICAL MATERIALS

Material	Density	Melting Point,	Specific Heat		Thermal Conductivity		Mean Coefficient of Thermal Expansion	
	lb/in.³	°F	Temp. Range, °F	Btu/lb per °F	Temp. Range, °F	Btu-in. / ft² hr °F	Temp. Range, °F	10^{-6}/°F
2024-T3 Clad Aluminum Alloy	0.100	935-1180						
7075-T6 Aluminum Alloy	.100	890-1180		(a)		(b)		(c)
FS1-H24 Magnesium Alloy	.064	1160		(a)		(b)		(c)
6Al-4V Titanium Alloy	.160	2800					90-932 932-1652	5.2 5.8
C-110M Titanium Alloy	.171	2730-2910		(a)		(b)		(c)
Inconel X Nickel Base	.298	2540		(a)		(b)		(c)
4340, 8630 Alloy Steel (200,000 psi T. S.)	.283	2500		(a)		(b)		(c)
Molybdenum	.369	4710		(a)		(b)		(c)
Tungsten	.697	6120	70-212 70-1832	0.034 .0365	70	1060	32-932 1112-1832	2.5 2.88
Aluminum Oxide Alite 212	.131	3720		(a)		(b)		(c)
Beryllium Oxide	.113	4500	257-1870	.071	2012	70-122	70-1800	5.0
Zirconium Boride	.180-.191	5400	60 1800	.115 .25	110	191	75-1000 75-2200	2.7 3.2
Silicon Carbide	.183	3990	70-1832 32-2550	.186 .285	2000 2200	66 109-113	70-2000 70-2550	2.3 2.44
Molybdenum Disilicide	.220	3400			300 1000 1400	385 240 220	70-2732	5.1
Titanium Carbide +10 to 40% Ni or Ni Alloy	.198-.234	5800			70	206-247	70-1200	4.5-6.2
Copper	.324	1980		(a)	212-387 212-1535	(b) 3040 2490		(c)
Beryllium	.066	2355		(a)		(b)		(c)
Niobium	.311	4370	32-212	.064			32-212	4.0

(a) see Figure 2-20. (b) see Figure 2-19. (c) see Figure 2-21.

TABLE 2-7. APPROXIMATE COSTS OF STRUCTURAL MATERIALS (1955)
(REFERENCE 43)

Material	Cost/pound
Aluminum Alloys	$ .51
Magnesium Alloys	.59
Titanium Alloys	15.00
Alloy Steels	
4130, 8630, 4340	.06
Stainless Steels	
18–18	.36
310	.60
430, 446	.60
17–7PH	.36
Highly Alloyed Austenitic Steels	
19–9DL	.75
N–155	4.00
S–590	3.43
S–816	6.14
A–286	1.87
Nickel Base Alloys	
Inconel X	2.00
Hastelloy X	2.50
Hastelloy C	3.50
Cobalt Base Alloys	
H. S. 25 (L–605)	5.50

TABLE 2-8. MECHANICAL PROPERTIES OF 2024-T6 ALUMINUM ALLOY PRODUCTS AT VARIOUS TEMPERATURES AND AFTER HEATING

(Representative Values—Not Guaranteed Minimums)

Max. or Min. Temp. °F	Tensile Properties							Stress, Time Relations for Rupture and Creep						Fatigue Strength at Temp. Indicated	
	Time at Temp. hr.	At Temperature Indicated			At Room Temperature After Heating			Time under stress hr.	10^3 psi to Rupture	10^3 psi for creep of				10^6 Cycles	Stress 10^3 psi
		Tensile Strength 10^3 psi	Yield Strength 10^3 psi	Elong. in 4D %	Tensile Strength 10^3 psi	Yield Strength 10^3 psi	Elong. in 4D %			1%	0.5%	0.2%	0.1%		
−320		83	68	11											
−112		72	60	10											
− 18		71	58	10											
75		69	57	10	69	57	10							0.1	43
														1.0	32
														10.0	24
														100.0	20
														500.0	18.5
212	0.5	65	55	10	69	57	10	0.1	63						
	10	65	55	10	69	57	10	1	60						
	100	65	55	10	69	57	10	10	57						
	1000	65	55	10	70	58	10	100	54						
	10000	65	55	10	70	60	10	1000	52						
300	0.5	60	52	11	69	57	10	0.1	56	54	52	50	47		
	10	60	52	12	69	58	10	1	53	51	48	48	41		
	100	59	51	13	69	58	10	10	49	48	46	41	33		
	1000	54	46	14	62	49	10	100	44	43	41	35	27		
	10000	45	36	17	53	37	10	1000	38	37	35	30	22		
400	0.5	50	43	13	67	53	10	0.1	49	46	43	39	34		
	10	44	37	16	53	43	10	1	43	41	38	33	24		
	100	38	31	19	51	36	10	10	35	33	30	23	15		
	1000	32	25	23	49	30	10	100	25	23	20	15	9		
	10000	27	20	27	42	24	11	1000	18	16	14	9.5	5.5		
500	0.5	30	25	18	54	38	10	0.1							
	10	26	21	22	49	31	10	1	26	22	19	15	11		
	100	20	16	28	44	23	12	10	17	15	13	10			
	1000	15	11	40	37	16	15	100	11	10	9				
	10000	12	9	55	28	10	20	1000	7						
600	0.5	17	14	30	47	27	10	0.1	17	16	15	10	6.5		
	10	12	9.5	45	39	18	13	1	12	10	8.5	5	3.5		
	100	10	7.5	55	35	13	17	10	7.5	6	4.5	3	2		
	1000	8.5	6	65	31	10	20	100	4.5	3.5	2.5	1.5			
	10000	8	6	75	27	9	20	1000	3	2	1.5				
700	0.5	8.5	6.5	55	40	19	13								
	10	6.5	5	70	37	12	18								
	100	5.5	4	85	34	10	20								
	1000	5.5	4	95	30	9	20								
	10000	5	4	100	27	9	20								

Courtesy ALCOA Research Laboratories

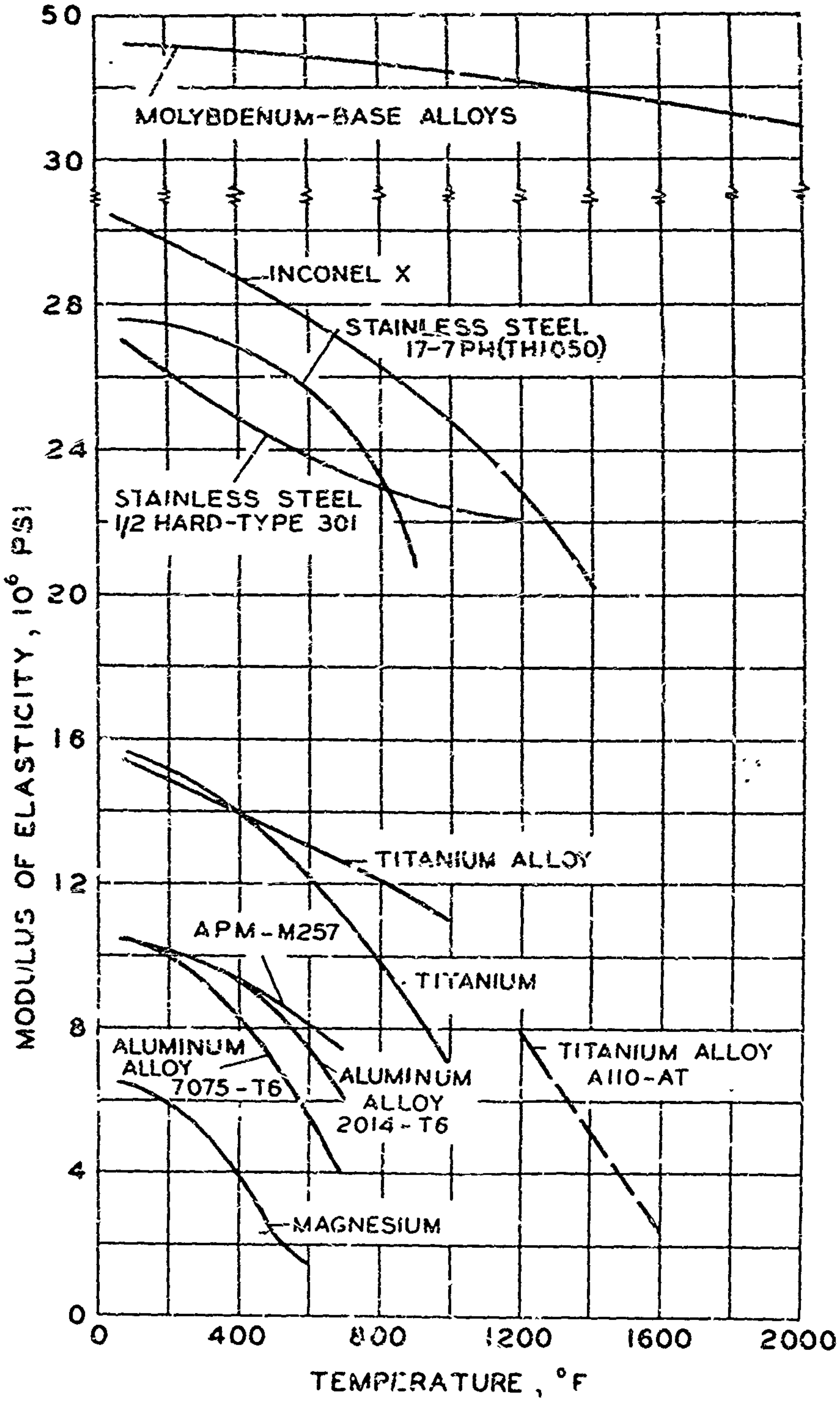


Figure 2-5. Effect of Temperature on Modulus of Elasticity for Several Typical Materials

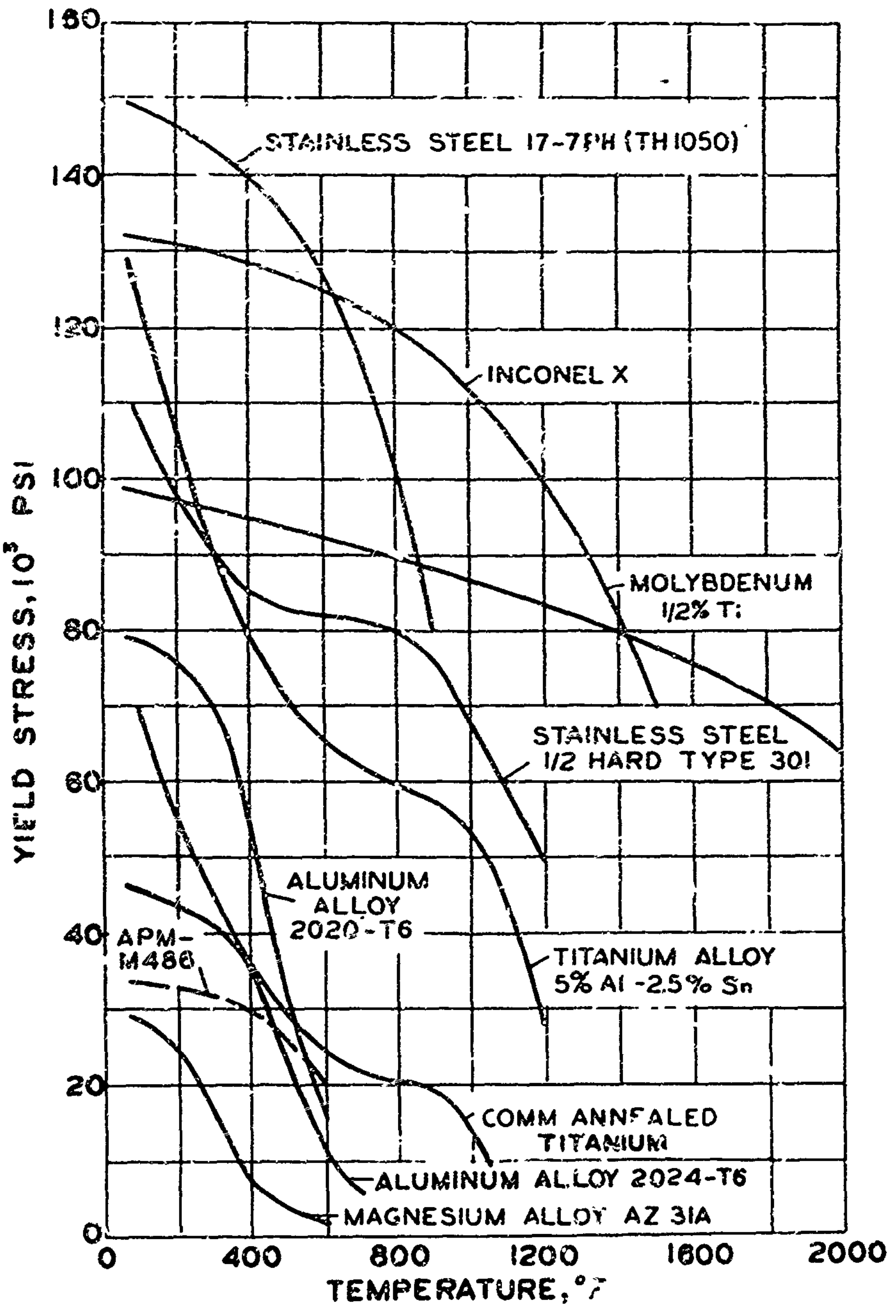


Figure 2-6. Effect of Temperature on Yield Stress for Several Typical Materials

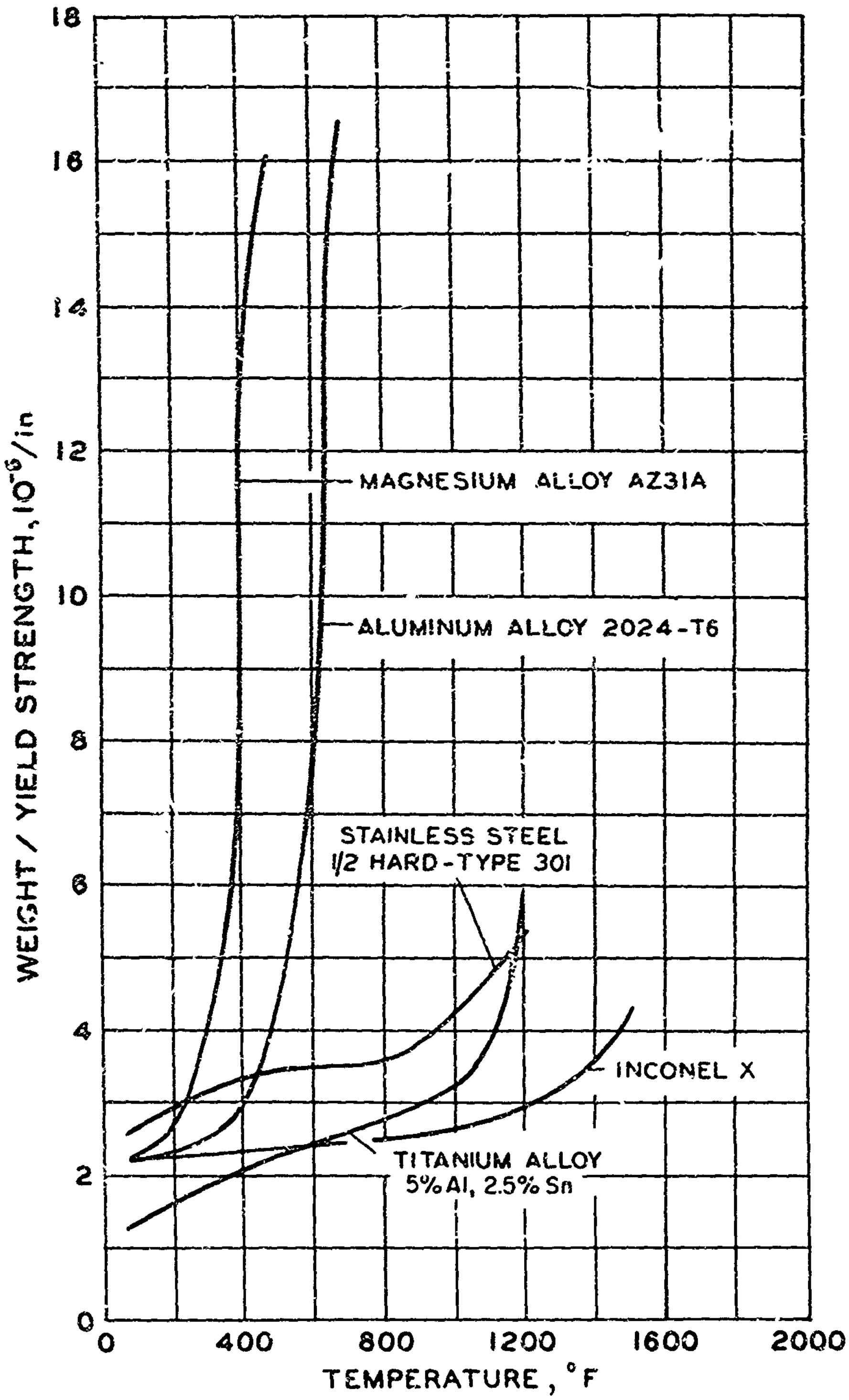


Figure 2-7. Effect of temperature on Weight-Strength Ratio for Several Typical Materials

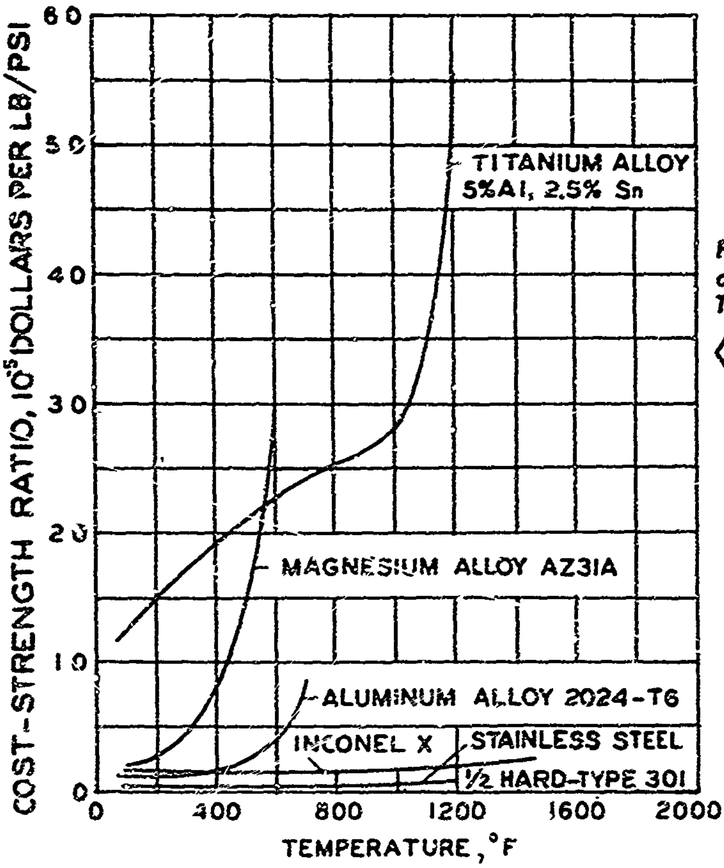
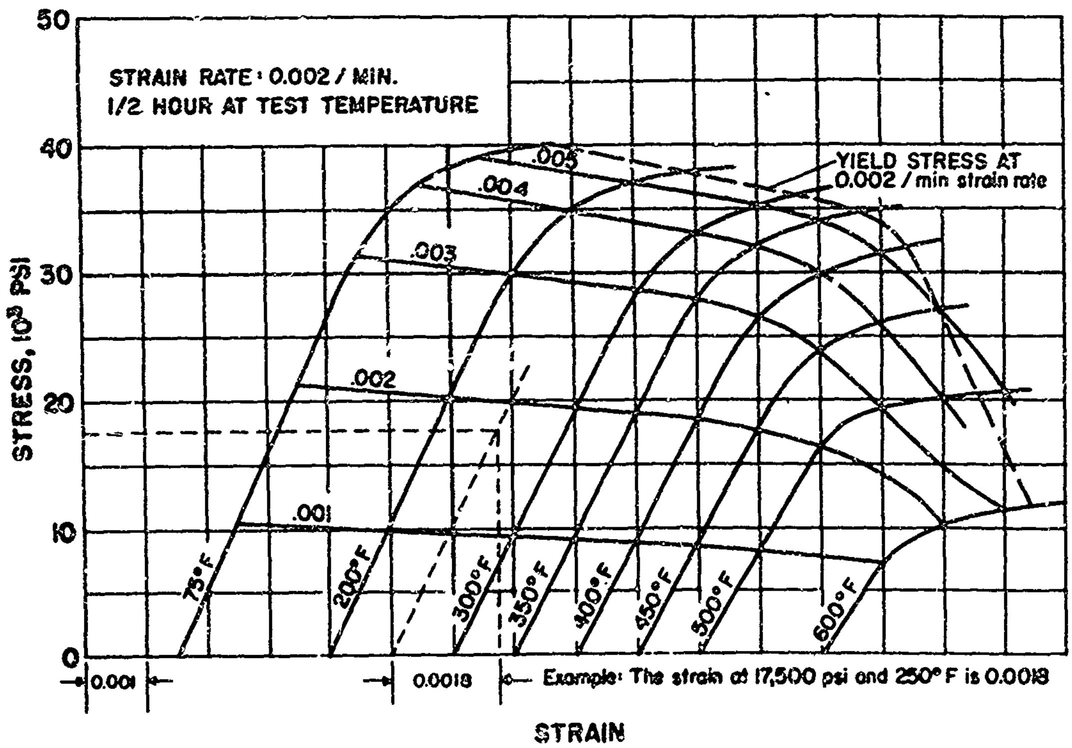


Figure 2-8. Effect of Temperature on Cost-Strength Ratio for Several Typical Materials

Figure 2-9. Compressive Stress-Strain Curves for 2024-T3 Aluminum Alloy Clad Sheet Stock at Various Temperatures



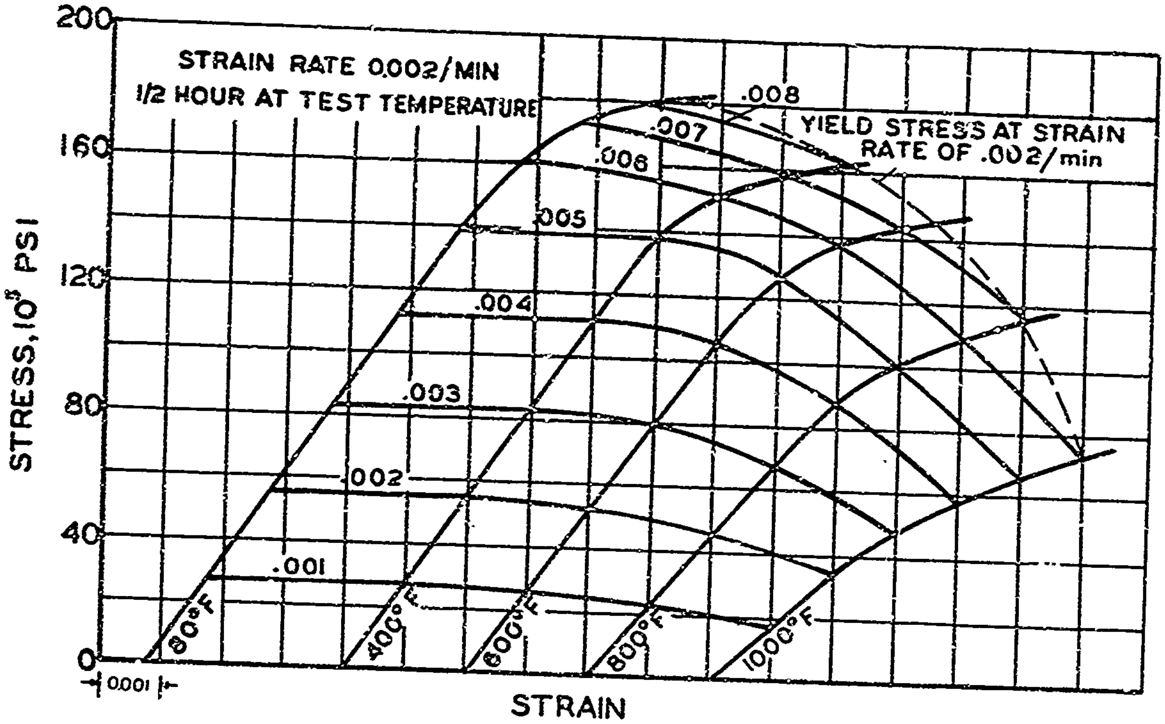
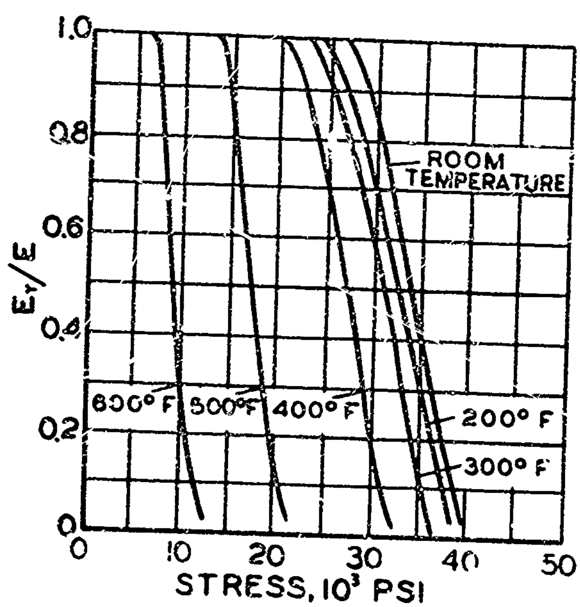


Figure 2-10. Compressive Stress-Strain Curves for 17-7 PH (TH 950) Stainless Steel Sheet Stock at Various Temperatures



TEMPERATURE °F	YOUNG'S MODULUS 10^6 PSI
ROOM TEMP.	10.7
200	10.3
300	9.9
400	9.4
500	8.8
600	7.9

Figure 2-11. Tangent Modulus Curves for 2024-T3 Aluminum Alloy Alclad Sheet in Compression

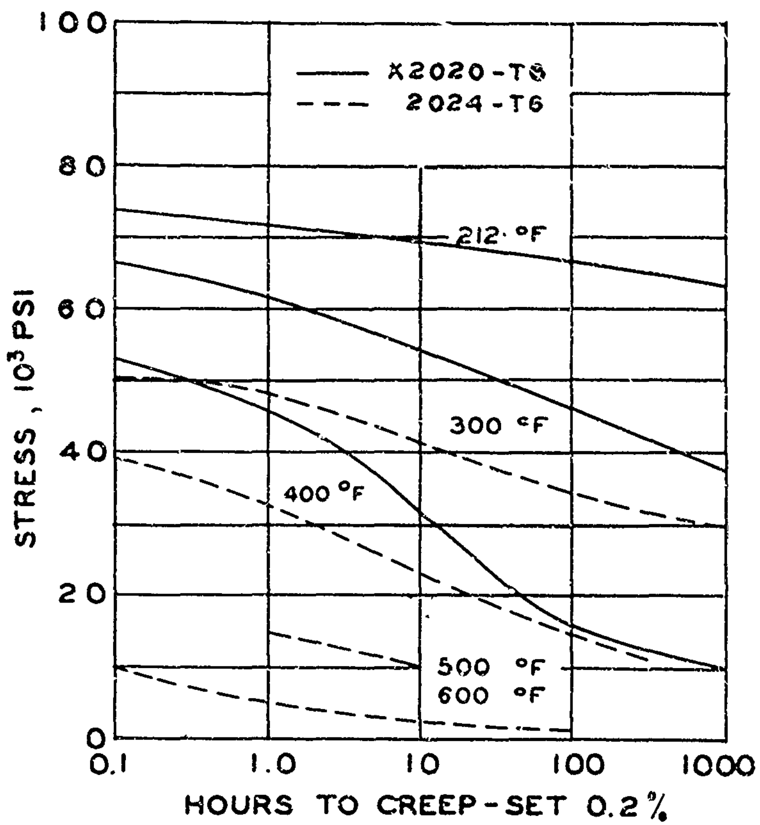


Figure 2-12. Stress for 0.2% Creep-Set of X2020-T6 and 2024-T6 Aluminum Alloys as a Function of Time and Temperature

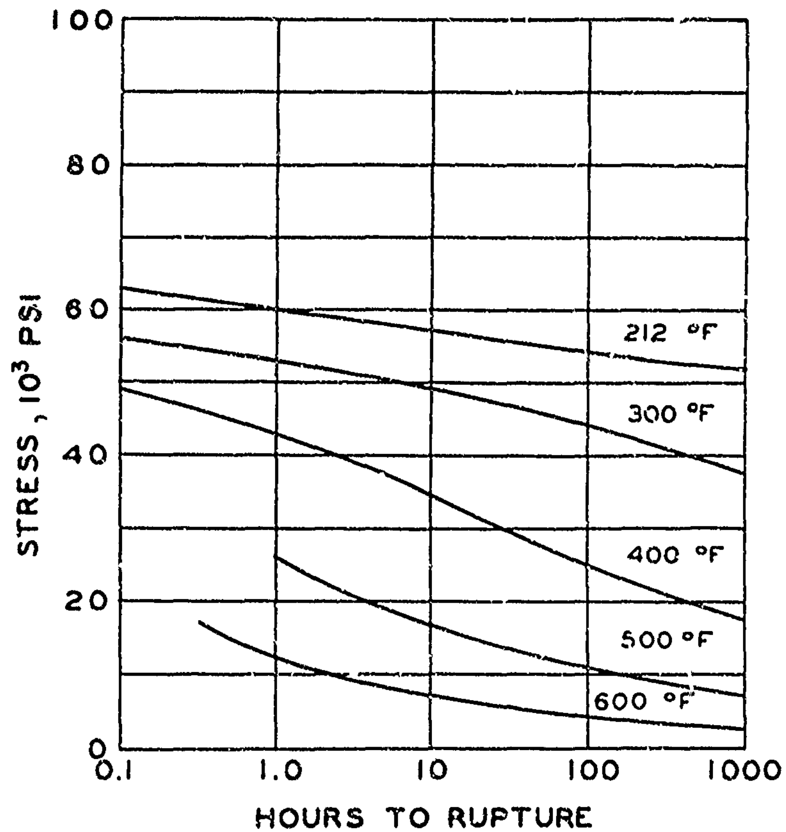
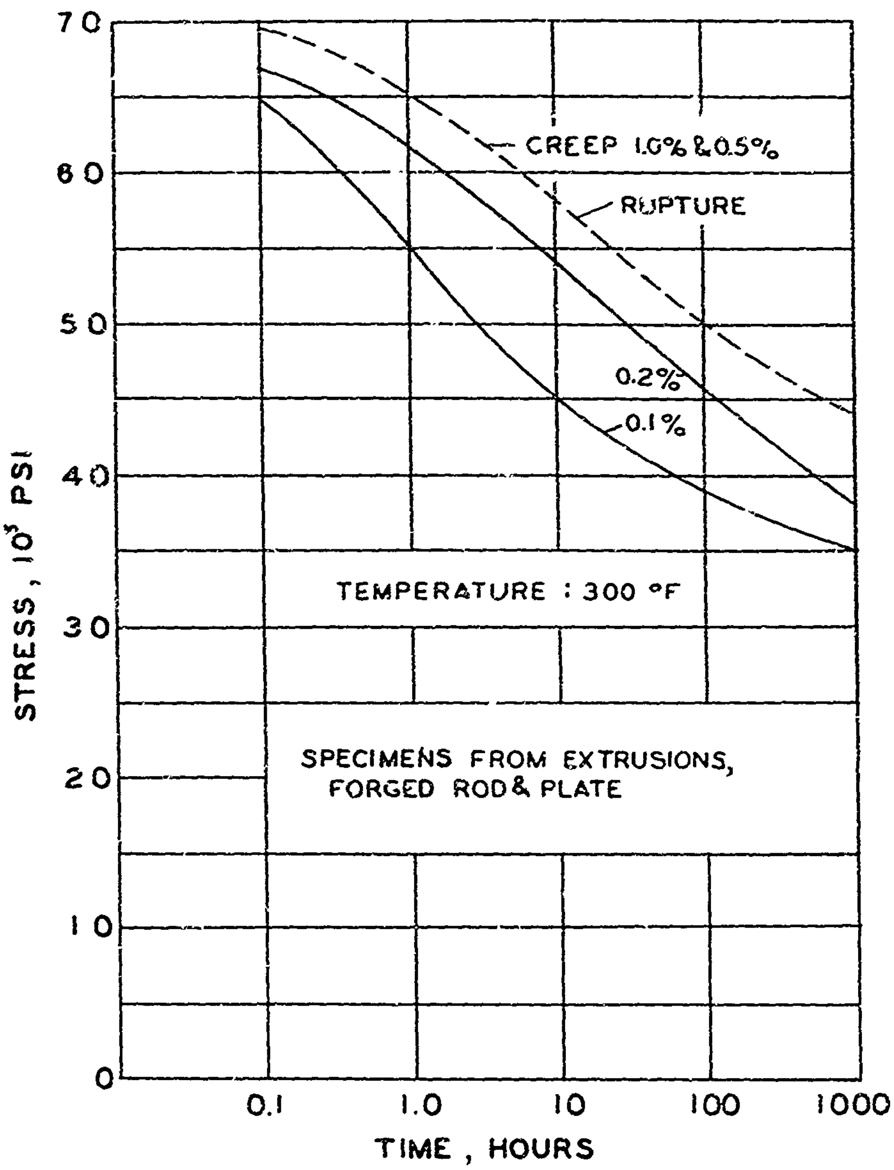


Figure 2-13. Rupture Stress for 2024-T6 Aluminum Alloy as a Function of Time and Temperature

Courtesy ALCOA Research Laboratories



Courtesy ALCOA Research Laboratories

Figure 2-14. Stress-Rupture and Creep Characteristics of X2020 Aluminum Alloy at 300°F

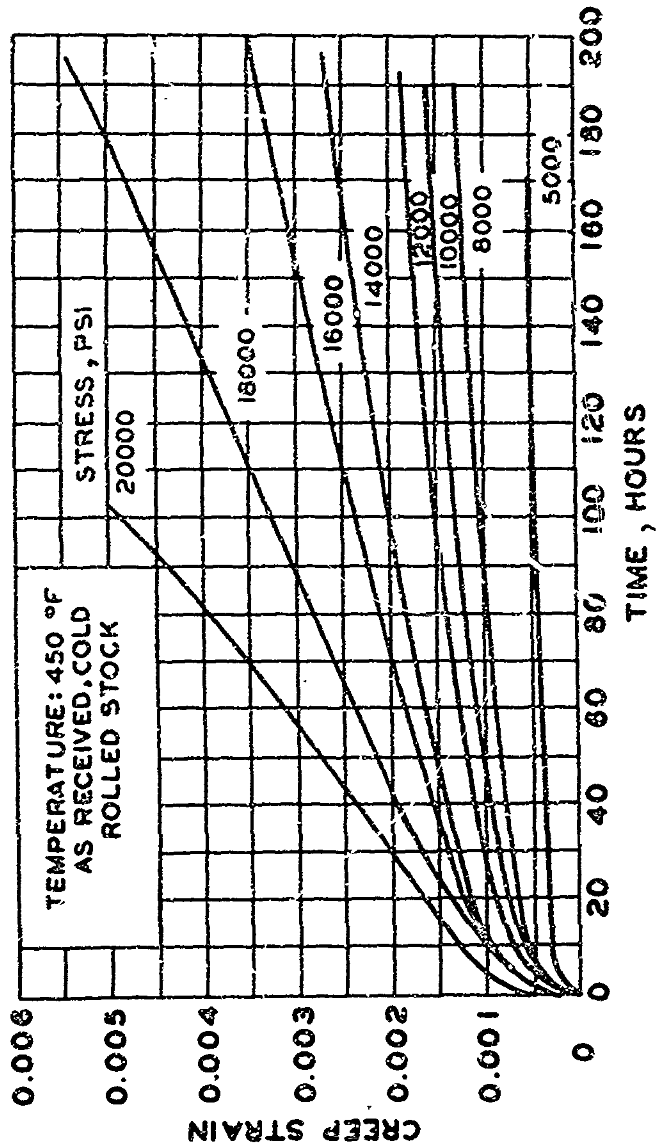
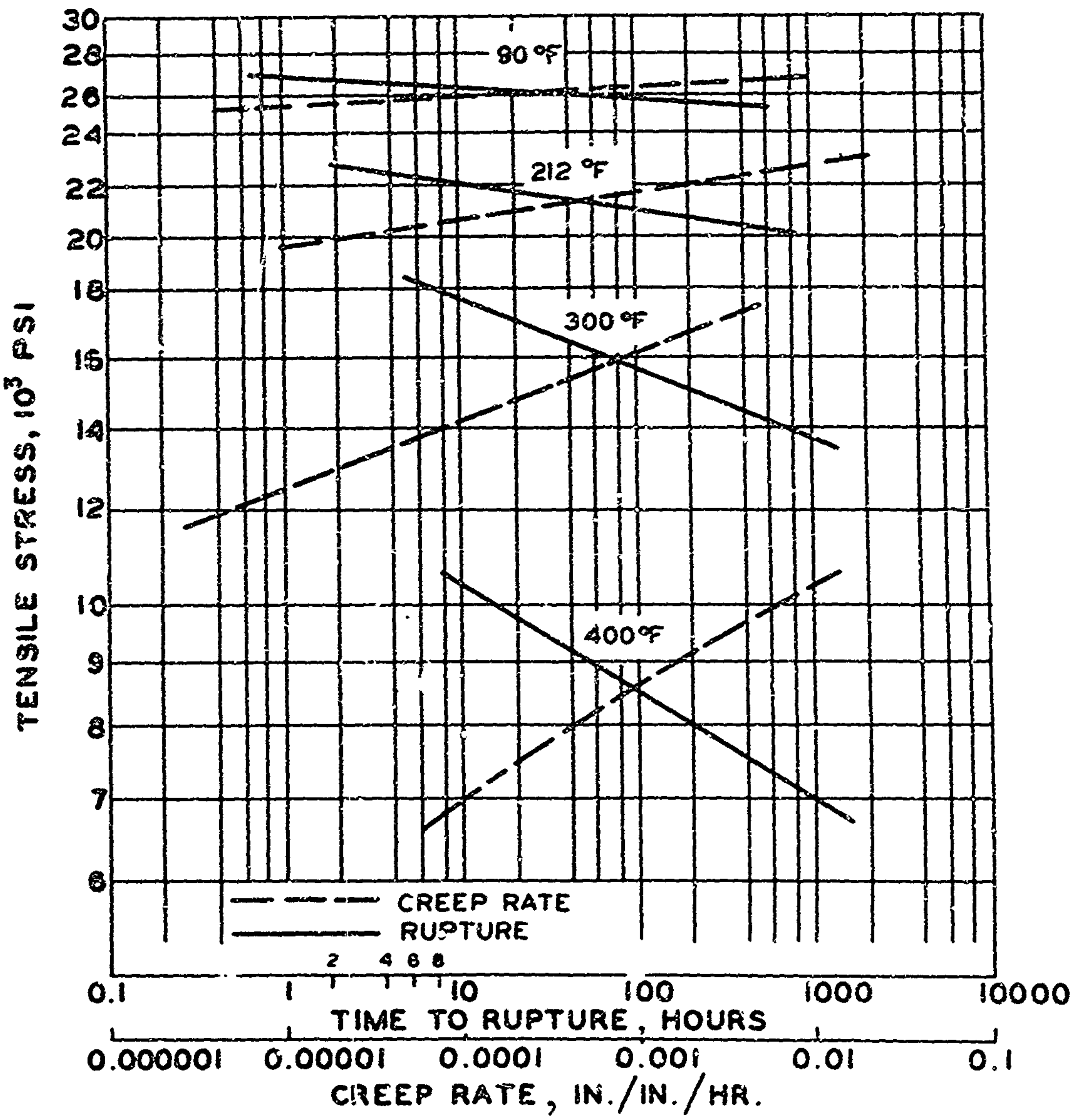
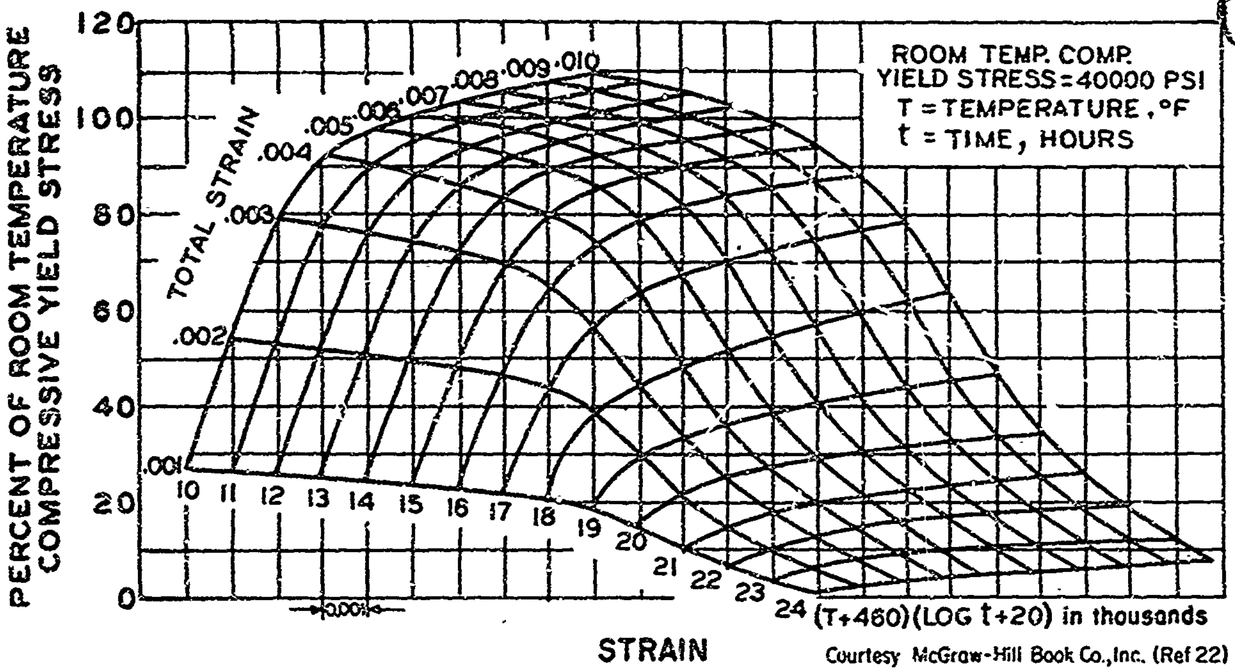


Figure 2-15. Tension Creep Curves for 2024-T4 Aluminum Alloy

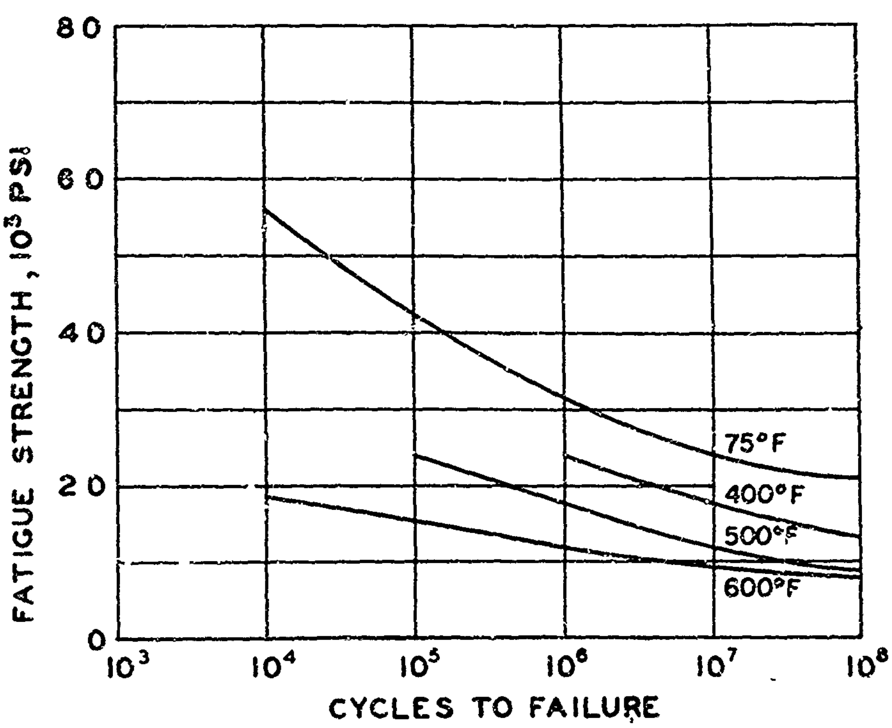


2-16. Creep Rate and Time to Rupture Curves for 3S-H18 Aluminum Alloy



Courtesy McGraw-Hill Book Co., Inc. (Ref 22)

Figure 2-17. Master Creep Curves for 2024-T3 Aluminum Alloy Clad Sheet Stock



Courtesy Alcoa Research Laboratories

Figure 2-18. Fatigue Properties of 2024-T4 Aluminum Alloy (Extruded Bar) at Elevated Temperatures

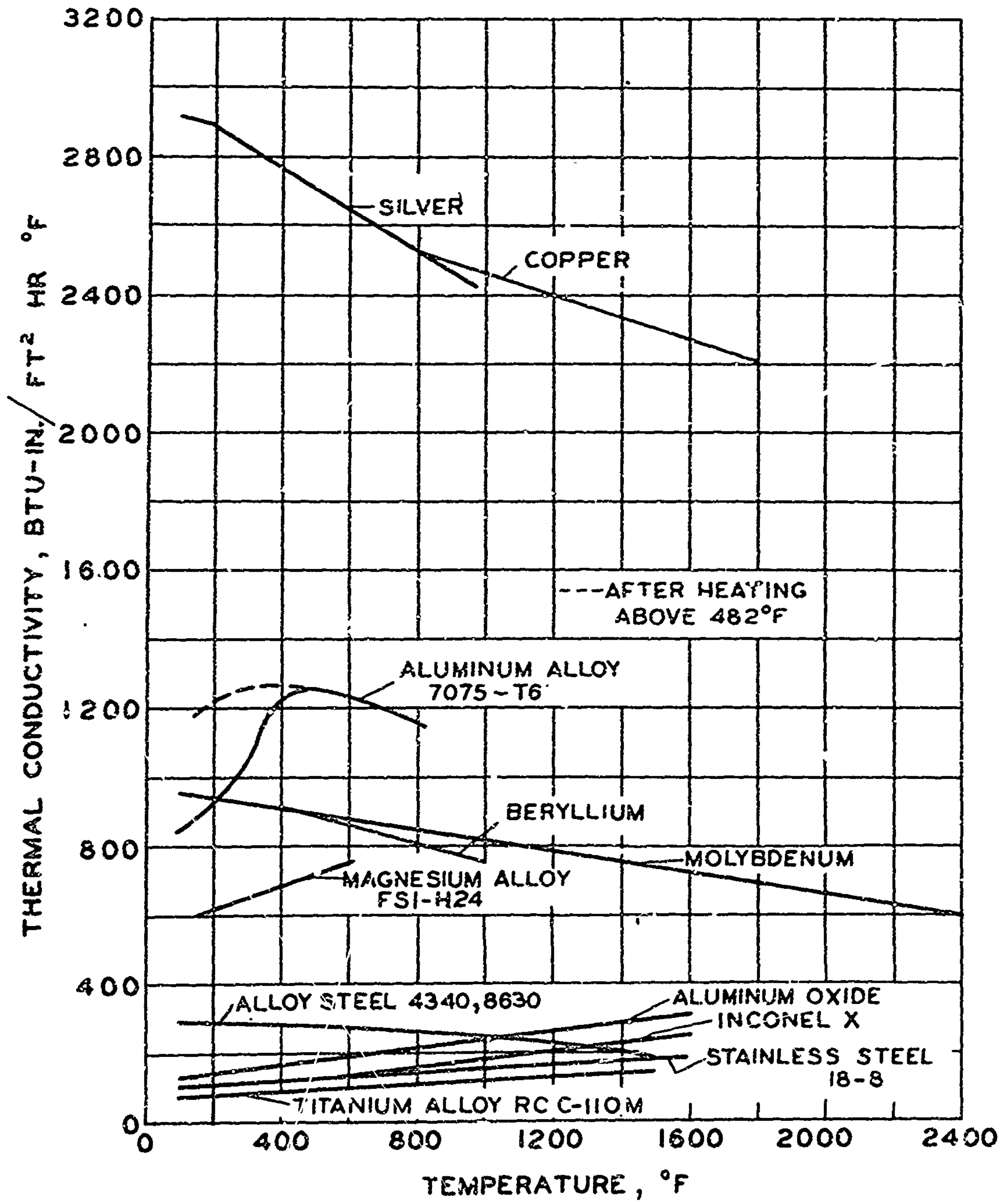


Figure 2-19. Thermal Conductivity of Selected Materials

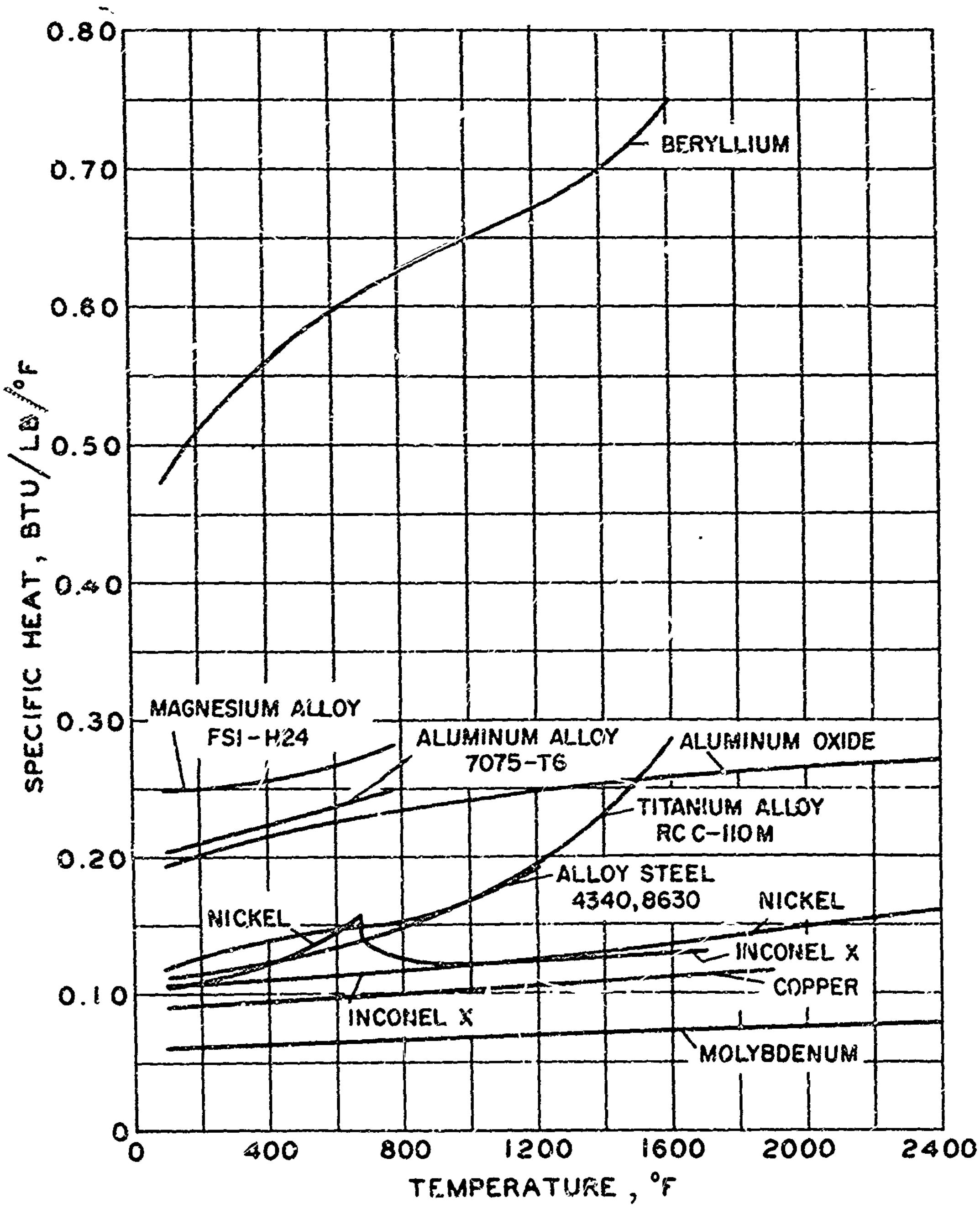


Figure 2-20. Specific Heat of Selected Materials

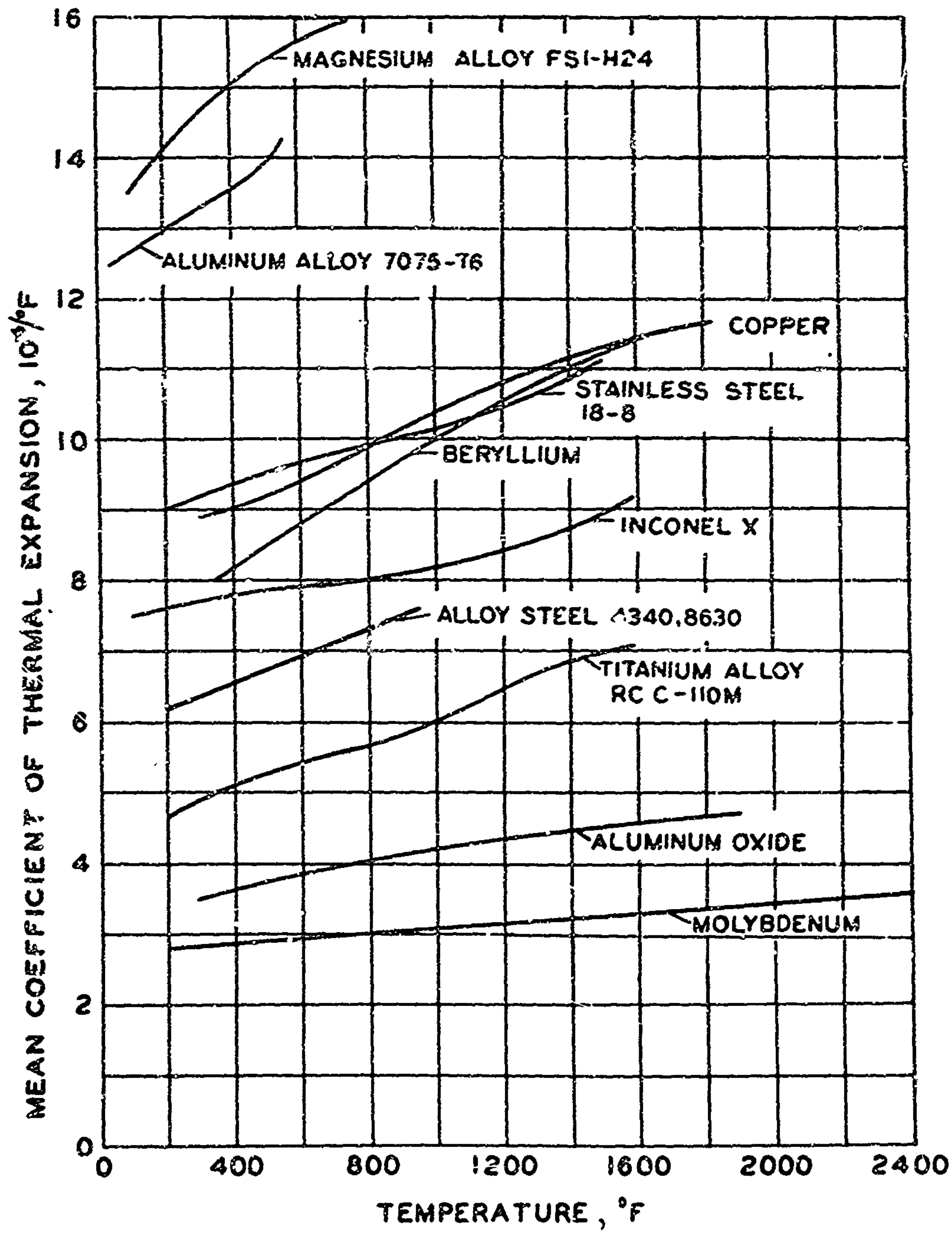


Figure 2-21. Mean Coefficient of Thermal Expansion for Several Materials

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Chapter 3

STRESS ANALYSIS

3-1. GENERAL CONSIDERATIONS

In the fifty-odd years since the first successful flight of a heavier-than-air machine, the analysis of the stresses in aircraft structures has been developed to a highly perfected art. Good textbooks on airplane stress analysis are readily available. The purpose of this chapter is to present those principles and formulas which are peculiar to missile structural analysis and are not to be found, as a rule, in books on airplane structures.

In this category belong shell analysis, the study of thermal stresses, and the investigation of the effects of creep. Shell analysis is peculiar to ballistic missile design because most of the structural elements of these missiles are thin shells, often unreinforced. The customary methods of reinforced thin shell analysis, amply discussed in textbooks on aircraft structures, are not applicable to these missiles. Thermal stresses and creep arise as a consequence of aerodynamic heating; for this reason they are of great importance to very-high-speed missiles but of little account in conventional airplanes.

3-2. STRESSES AND DEFORMATIONS IN THIN SHELLS UNDER UNIFORM INTERNAL PRESSURE

3-2.1. Circular Cylindrical Shell. The *tangential* or *hoop stress*, σ_t, and the stress in the *longitudinal* or *axial* direction, σ_x, within circular cylindrical shells under a uniform internal pressure are

$$\sigma_t = \frac{pR}{h} \tag{3-1}$$

$$\sigma_x = \frac{pR}{2h} \tag{3-2}$$

where p is the excess of the internal pressure in the closed cylindrical shell over the external pressure, R is the mean radius of the cylindrical shell, and h is the wall thickness. The stresses σ_t and σ_x are tensile if the internal pressure is larger than the external pressure, and compressive when the opposite is true.

3-2.2. Conical Shell. In a conical shell under a uniform internal pressure the tangential stress, σ_t, and the axial stress acting in the direction of the generator, σ_x, are

$$\sigma_t = pR/h \cos\gamma \tag{3-3}$$

$$\sigma_x = pR/2h \cos\gamma \tag{3-4}$$

where R is the local mean radius of the shell measured in a plane perpendicular to the axis of the cone, γ is the angle subtended by any generator with the axis of the cone (the half apex angle of the cone), and the remaining symbols are defined as before.

3-2.3. Spherical Shell. The normal stress acting in any direction in the plane tangential to the sphere at the considered point known as the membrane stress, σ_m, may be expressed as

$$\sigma_m = \frac{pR}{2h} \tag{3-5}$$

3-2.4. Thin Shell. All of the stresses given heretofore are membrane stresses. These are stresses distributed uniformly through the thickness of the wall. Under their action the shell changes its size slightly. When the material is perfectly elastic and when it follows Hooke's uniaxial law

$$\epsilon = \sigma/E \tag{3-6}$$

where ϵ is the strain (elongation per unit length) and E Young's Modulus of elasticity, the mean radius of the cylinder increases by the amount

$$\Delta R_{cyl} = \left(1 - \frac{\nu}{2}\right)\frac{pR^2}{Eh} \tag{3-7}$$

where ν is Poisson's ratio whose value is 0.3 for steel and aluminum alloys. For the cone the corresponding quantity is

$$\Delta R_{cone} = \left(1 - \frac{\nu}{2}\right)\frac{pR^2}{Eh\cos\gamma} \tag{3-8}$$

and for the sphere

$$\Delta R_{sphere} = (1 - \nu)\frac{pR^2}{2Eh} \tag{3-9}$$

3-3. DISCONTINUITY STRESSES IN THIN SHELLS

When a cylindrical pressure vessel is provided with two hemispherical pressure vessel heads, and the vessel is filled with a gas at high pressure, the change in the length of the radius of the cylinder is more than twice the change in the length of the radius of the sphere in accordance with Equations (3-7) and (3-9) if the wall thicknesses are equal. Such deformations are obviously impossible when cylinder and hemisphere are welded together (see Figure 3-1). For this reason bending stresses develop in the walls of the vessel in addition to the membrane stresses discussed in Section 3-2. When the material is perfectly elastic, these bending stresses vary linearly across the wall and attain their maximum values on the outer and inner surfaces of the wall. The bending stresses add up to a bending moment of M_o per unit length of the circumference at the edge of the cylinder of Figure 3-2 and the shearing stresses accompanying them to a shear force V_o per unit length of the circumference.

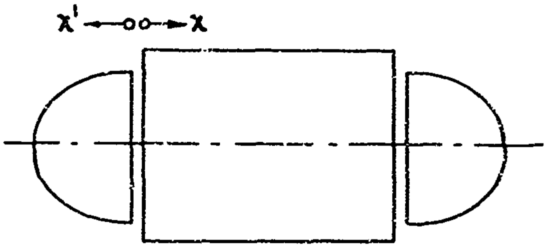

Figure 3-1. Theoretical Deformations of Pressure Vessel under Membrane Stresses Alone

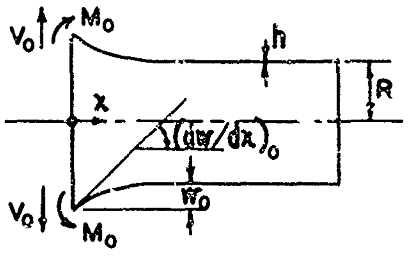

Figure 3-2. Deformations of Thin Circular Cylindrical Shell under Bending Stresses

Under the action of the edge moment and the edge shear force the region of the shell close to the edge deforms. The deformations of the edges of cylinder and hemisphere are of such magnitude that the discontinuity in the deformations as caused by the membrane stresses alone is eliminated and cylinder and hemisphere fit together. The bending stresses caused by M_o and V_o are known as *discontinuity stresses.*

When M_o and V_o are applied to the edge of the cylindrical shell shown in Figure 3-2, the *radial displacement* caused by them is given by the following equation

$$w_o = \frac{2\sqrt{3(1-\nu^2)}}{Eh}\left(\frac{R}{h}\right)\left(M_o + \frac{V_oR}{K}\right) \qquad (3\text{-}10)$$

The positive directions of M, V and w are as shown in the figure. The slope of the deformed shape of the shell is

$$(dw/dx)_o = \frac{-2\sqrt{3(1-\nu^2)}}{Eh^2}(2M_oK + V_oR) \qquad (3\text{-}11)$$

Equation (3-11) yields a negative slope when M_o and V_o are positive; correspondingly the radial displacement w decreases with increasing x in Figure 3-2, that is the slope shown in the figure is negative.

The value of the constant K in Equations (3-10) and (3-11) is

$$K = \sqrt[4]{3(1-\nu^2)}\sqrt{(R/h)} \qquad (3\text{-}12)$$

The bending moment M_o is not necessarily the largest bending moment in the wall of the shell; variation in the value of the bending moment is given by the equation

$$M = e^{-Kx/R}\left[M_o\left(\cos\frac{Kx}{R} + \sin\frac{Kx}{R}\right) + \frac{V_oR}{K}\sin\frac{Kx}{R}\right] \qquad (3\text{-}13)$$

The location of the maximum bending moment can be computed from the formula

$$\cot\frac{Kx}{R} = 1 + \frac{2M_oK}{V_oR} \qquad (3\text{-}14)$$

The connection between the maximum bending stress and the bending moment is

$$\sigma_{b_{max}} = 6M/h^2 \qquad (3\text{-}15)$$

The attenuation of the bending stresses is characterized by the factor $e^{-Kx/R}$; this is unity at $x = 0$ and $e^{-1} = 0.368$ when

$$x_{at} = R/K \qquad (3\text{-}16)$$

When the attenuation length x_{at} is small as compared to the undisturbed, uniform portion of the length of the cylinder, the equations given here are valid; if the attenuation length is larger the equations must be replaced by more complex equations discussed in the literature.

Experience has shown that satisfactory accuracy is obtained for engineering purposes if the actual shell configurations are imagined to be replaced by equivalent circular cylindrical shells in the calculation of the disturbance stresses. In the case of the spherical shell, the equivalent cylindrical shell has the same radius as the spherical shell that it replaces. In the case of the arbitrary rotationally symmetric shell which joins the cylinder with a continuous tangent, the radius of the equivalent shell is

$$R_{eq} = R(r/R)^2 \qquad (3\text{–}17)$$

where R is the radius of the cylinder and r is the radius of the meridian of the rotationally symmetric shell at the junction with the cylinder. Equations (3-10) to (3-16) retain their validity for the general rotationally symmetric shell joining the cylinder with a continuous tangent if R is replaced in them by R_{eq} and if x is measured along a meridian. This is true, however, only if the change in the values of the principal curvatures of the rotationally symmetric shell is insignificant over the attenuation length x_{at} defined in Equation (3-16).

When the head of the pressure vessel has a tangent which changes discontinuously from the tangent of the cylinder, the equivalent cylinder concept is still valid, but the forces and displacements must be resolved in the proper directions.

For illustrative purposes the numerical values characterizing the pressure vessel of Figure 3-1 may be taken as

Radius of median surface of cylinder

$R_{cyl} = 25$ in

Radius of median surface of hemisphere

$R_{sphere} = 25$ in

Length of cylinder

$L = 100$ in

Wall thickness of cylinder and hemisphere

$h = 0.125$ in

Modulus of elasticity

$E = 29 \times 10^6$ psi

Poisson's ratio

$\nu = 0.3$

Internal pressure

$p = 100$ psi

From Equations (3–1), (3–2) and (3–5) one obtains

Hoop stress in cylinder

$\sigma_t = 20{,}000$ psi

Axial stress in cylinder

$\sigma_x = 10{,}000$ psi

Membrane stress in hemisphere

$\sigma_m = 10{,}000$ psi

The changes in the length of the radius of cylinder and hemisphere are computed from Equations (3–7) and (3–9)

$$\Delta R_{cyl} = 1.466 \times 10^{-2} \text{ in}$$

$$\Delta R_{sphere} = 0.603 \times 10^{-2} \text{ in}$$

The misfit between cylinder and hemisphere is therefore

$$\Delta R_{cyl} - \Delta R_{sphere} = 0.863 \times 10^{-2} \text{ in}$$

This misfit must be eliminated with the aid of the bending stresses.

First, the hemisphere is replaced by the equivalent cylinder, and a coordinate x^1 is introduced in it which is measured to the left from the juncture between cylinder and hemisphere. An edge moment M_o and an edge shear force V_o are applied to the cylinder as shown in Figure 3-2 and equal and opposite quantities are applied to the cylinder replacing the hemisphere. The conditions must now be enforced that under the action of M_o and V_o the relative displacement of the two shell walls is 0.863×10^{-2} in., and that the slope of the generators remains continuous. The latter condition is fulfilled if $M_o = 0$ because the wall thicknesses are the same in cylinder and hemisphere. As the real and the equivalent cylinder are equal in size, under the action of the same shear force V_o each one shows the same numerical value of change in the length of the radius. Hence Equation (3-10) becomes

$$V_o = \frac{-EK}{\sqrt[4]{3(1-\nu^2)}}\left(\frac{h}{R}\right)^2(\Delta R_{cyl} - \Delta R_{sphere}) \qquad (3\text{–}18)$$

From Equation (3-12) $K = 18.2$ and substitution of the numerical values in Equation (3-18) yields

$$V_o = -1.99 \times 10^3 \times 8.63 \times 10^{-3}$$
$$= -17.2 \text{ lb per in}$$

With $M_o = 0$ the condition of a maximum moment becomes

$$Kx/R = \pi/4 \qquad (3\text{-}19)$$

Substitution in Equation (3-13) leads to

$$M_{max} = 0.322 V_o R/K = 0.442 V_o$$
$$= -7.6 \text{ in lb per in}$$

The corresponding stress is from Equation (3-15)

$$\sigma_{b\,max} = 2920 \text{ psi}$$

From Equation (3-19) the location of the maximum bending stress is

$$x = (\pi/4)(R/K) = 1.08 \text{ in.}$$

which is very close to the juncture. The attenuation length is from Equation (3-16)

$$x_{at} = R/K = 1.37 \text{ in.}$$

Hence interference from the top of the hemisphere and from the other end of the cylinder is negligible and the results obtained are accurate enough for engineering purposes.

Superposition of σ_x and of $\sigma_{b\,max}$ yields the maximum normal stress which acts in the cylinder in the axial direction at a distance of 1.08 in. from the juncture on the outer surface of the wall:

$$\sigma_{max.\,max.} = 10{,}000 + 2920 = 12{,}920 \text{ psi}$$

Finally a general formula can be derived from Equations (3-13), (3-18) and (3-19) which is valid when the cylinder and the equivalent cylinder have the same radius and the same wall thickness:

$$M_{max} = -0.0465 \frac{E}{\sqrt{1-\nu^2}} h^2 \frac{\Delta R_{cyl} - \Delta R_{sphere}}{R} \qquad (3\text{-}20)$$

When $\nu = 0.3$, this becomes

$$M_{max} = -0.0487 E h^2 \frac{\Delta R_{cyl} - \Delta R_{sphere}}{R} \qquad (3\text{-}21)$$

3-4. BUCKLING OF THIN CIRCULAR CYLINDRICAL SHELLS

3-4.1. Uniform Lateral Pressure. If a uniform external pressure p is acting over the circular cylindrical surface of a shell, but not over the end sections, the cylinder buckles when the pressure reaches the following critical value:

$$p_{cr} = 0.92E \frac{h}{R} \sqrt{\frac{h^2}{RL^2}} \qquad (3\text{-}22)$$

Here E is Young's Modulus, h the wall thickness, R the radius of the median surface of the cylinder, and L the length of the cylinder between supports; Poisson's ratio ν is taken as 0.3. This equation is valid only if

$$0.763 \sqrt{L^2/Rh} \geq 1 \qquad (3\text{-}23)$$

The meaning of this restriction should be understood from a numerical example. When $R = 25$ in. and $h = 1/8$ in., the inequality reduces to $0.4321 \geq 1$. If $L = 10$ in., the left-hand member is 4.32; with this value one can expect a reasonable approximation. The accuracy increases with increasing L.

For very long cylinders the critical pressure is

$$p_{cr} = [E/4(1-\nu^2)](h/R)^3 \qquad (3\text{-}24)$$

Equation (3-24) is to be used in preference to Equation (3-22) when it predicts a higher buckling pressure than Equation (3-22).

For very short cylinders which do not satisfy inequality (3-23), the literature should be consulted.[1,2,3]

3-4.2. Uniform Axial Pressure. The buckling stress under the application of a uniform axial pressure can be calculated from the formula [4]

$$\sigma_{cr} = kE(h/R) \qquad (3\text{-}25)$$

where

$$k = 9(h/R)^{0.6} + 0.16(R/L)^{1.3}(h/R)^{0.3} \qquad (3\text{-}26)$$

and h, R, and L are as defined in the previous paragraph. For example, $k = 0.24$ when $R/h = 500$ and $R/L = 1$. For short and comparatively thick-walled cylinders Equation (3-25) yields values of k exceeding 0.3; it is recommended that in such cases k should be taken as 0.3.

3-4.3. Pure Bending. It is customary to calculate the maximum bending stress in a cylindrical shell from the engineering theory of bending. If the maximum compressive stress so found is less than 1.33 times the critical axial stress obtained from Equation (3-24), the cylinder is not expected to buckle.

3-4.4. Pure Torsion. The critical shear stress τ_{cr} in pure torsion is[5]

$$\tau_{cr} = E(h/L)^2\{3.08 + [3.15 + 0.557(L^2/hR)^{3/2}]^{1/2}\} \qquad (3\text{-}27)$$

if Poisson's ratio is taken as 0.3.[1,2]

For very long cylinders the following formula applies:

$$\tau_{cr} = 0.25\ E\ (h/R)^{3/2} \qquad (3\text{-}28)$$

3-4.5. Hydrostatic Pressure. In the hydrostatic case of loading, the cylinder is closed at its two ends and the closed shell is subjected to a uniform external pressure p. When the cylinder is very long, it buckles under the critical pressure corresponding to uniform lateral pressure; and when it is very short, under the critical stress of the axial loading case. In the range between these two extremes the critical pressure of the hydrostatic loading case is lower than the critical pressure of either of the simpler loading cases mentioned. For exact values, see the literature.[1,2] An approximate formula can be given as[6]

$$p_{cr} = \frac{0.92\ E\ (h/R)^2}{(L/R)(R/h)^{1/2} - 0.636} \qquad (3\text{-}29)$$

3-4.6. Combined Axial Compression and Torsion. An empirical formula[7] can be given in the form:

$$R_c + R_t^2 < 1 \qquad (3\text{-}30)$$

In this interaction formula R_c is the ratio of the compressive stress of the combined loading to the critical compressive stress under compression alone, and R_t is the ratio of the shear stress of the combined loading to the critical shear stress of torsion alone. When the inequality is satisfied, the cylindrical shell is safe. When the left-hand member of the inequality is equal to unity, the shell buckles. No equilibrium can be maintained when the left-hand member exceeds unity.

3-4.7. Different Boundary Conditions. The equations given above are valid for the case when the edges of the cylinder are simply supported. This boundary condition involves no constraint against rotation of the generators of the cylinder over the supports and no resistance to a warping of the end section of the cylinder out of its plane; however, radial and circumferential displacements of the circular median line of the wall are prevented in the end sections.

When the end section of the cylinder is welded or bolted to another structural element, warping of the plane and rotation of the generators may be partially or fully prevented. This additional constraint raises the critical stress but the increase is usually not significant except when the cylinder is very short. In the latter case more accurate values of the buckling stress can be taken from the literature.[1,2]

3-4.8. Stabilization Through Internal Pressure. Thin shells can be partially, or fully, stabilized[1] through application of an internal pressure p. The stabilizing effect in the case of a circular cylindrical shell subjected to a uniformly distributed axial compressive stress can be expressed by an addition of Δk to the value k of the factor in Equation (3-25). First calculate the non-dimensional pressure p^* defined as

$$p^* = (p/E)(R/h)^2 \qquad (3\text{-}31)$$

The increment Δk is then given by

$$\Delta k = 1.9\ p^* \text{ if } 0 \le p^* \le 0.12 \qquad (3\text{-}32)$$

$$\Delta k = 0.229 \text{ if } p^* > 0.12$$

Finally, Equation (3-24) is replaced by

$$\sigma_{cr} = (k + \Delta k)\ E\ (h/R) \qquad (3\text{-}33)$$

If the example given under Paragraph 3-4.2 is continued and an internal pressure p = 10 psi is applied to the shell, one obtains $p^* = 0.0862$ and $\Delta k = 0.164$. Hence the critical stress becomes:

$\sigma_{cr} = 0.24 \times 29 \times 10^6/500 = 13{,}900$ psi without internal pressure

$\sigma_{cr} = 0.404 \times 29 \times 10^6/500 = 23{,}450$ psi with internal pressure

Young's Modulus was assumed to be 29×10^6 psi (steel).

A limited amount of experimental evidence[8] indicates that Equation (3-30) can be used to determine the critical condition of a circular cylindrical shell subjected simultaneously to compression, torsion and internal pressure. It is only necessary to re-interpret the meaning of the symbols R_c and R_t. R_c is now the ratio of the applied compressive stress of the triple loading (compression plus torsion plus pressure) to the critical com-

pressive stress in the presence of the prescribed internal pressure p. Similarly, R_t is the ratio of the shearing stress caused by the torque of the triple loading to the critical shearing stress of torsion in the presence of the prescribed internal pressure p.

A circular cylindrical shell subjected simultaneously to torsion and internal pressure does not buckle if the following inequality is satisfied:[9,10]

$$R_t^2 - R_p < 1 \qquad (3\text{-}34)$$

where R_t is the ratio of the shearing stress of the combined loading to the critical shearing stress under pure torsion while R_p is the ratio of the internal pressure p to the critical pressure of hydrostatic loading (both pressures are considered positive in this context).

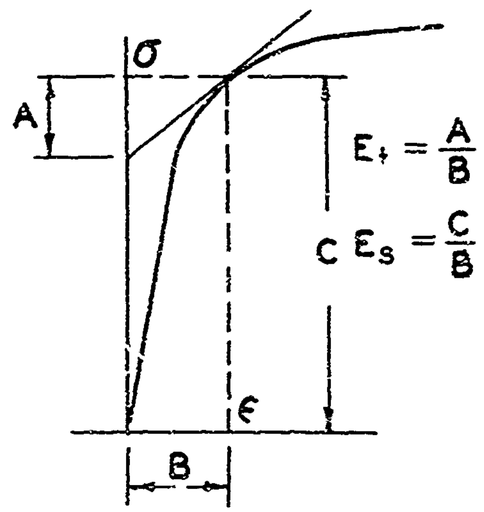

Figure 3-3. Determination of Tangent Modulus and Secant Modulus from Stress-Strain Curve

3-4.9. **Inelastic Buckling.** When the stresses at which buckling takes place according to the formulas given are higher than the limit of elasticity of the material, the theories underlying the formulas are not valid. They must be modified in order to arrive at results useful in engineering applications. As the theory of inelastic behavior is in lesser agreement with test results than the theory of elastic behavior, plastic buckling formulas must be used with some degree of caution.

In the calculation of buckling stresses greater than the elastic limit, two moduli are of importance in addition to Young's Modulus E. One is the tangent modulus E_t defined as the rate of change of the stress with strain at the particular value of the stress; and the other the secant modulus E_s which is the ratio of the particular value of the stress to the corresponding value of the strain (see Figure 3-3). Both E_t and E_s are equal to E when the stress-strain relation is represented by Hooke's straight line. In the case of plastic buckling, E in the formulas must be replaced by some combination of E_t and E_s; in a first approximation one may use E_s instead of E. For more accurate results recourse should be had to the literature.[11,12]

3-5. BUCKLING OF THIN SPHERICAL SHELLS

The critical compressive stress in a uniformly compressed thin-walled spherical shell is

$$\sigma_{cr} = 0.2E(h/R) \qquad (3\text{-}35)$$

The corresponding value of the critical pressure is

$$p_{cr} = 0.4E(h/R)^2 \qquad (3\text{-}36)$$

3-6. THERMAL STRESSES

3-6.1. **Straight Bar.** If a straight bar were heated from a uniform initial temperature T_i to a uniform final temperature T, it would expand and its initial length L would increase by the amount

$$\Delta L = \alpha(T - T_i)L \qquad (3\text{-}37)$$

where α is the coefficient of thermal expansion, usually measured in inches per inch per degree Fahrenheit.

If, however, the bar is held between insulated walls as shown in Figure 3-4, the insulated walls prevent this expansion completely by setting up thermal stresses of a magnitude

$$\sigma = -\alpha E(T - T_i) \qquad (3\text{-}38)$$

where E is Young's Modulus and the negative sign indicates that the thermal stress is

compressive if the final temperature T is higher than the initial temperature T_i. It is easy to prove that the shortening of the bar in consequence of this thermal stress is equal to the elongation caused by the rise in temperature as given by Equation (3-37).

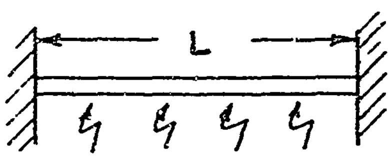

Figure 3-4. Bar Heated between Two Insulated Walls

3-6.2. **Flat Plate.** If a long rectangular flat plate of constant thickness h is heated from a uniform initial temperature T_i to a final variable temperature T, whose distribution is symmetric with respect to the x axis (see Figure 3-5), and if rigid end walls prevent all expansion, the thermal stress distribution is still given by Equation (3-38). In this case, T, as well as the thermal stress, is a function of the coordinate y while it is independent of the coordinate x. In the absence of rigid walls such normal stresses cannot exist at the free ends, x = ± L/2, the thermal stress is given by

$$\sigma = -\alpha E[(T - T_i) - (T - T_i)_{av}] \quad (3\text{-}39)$$

where the average rise in temperature is

$$(T + T_i)_{av} = (1/b)\int_{-b/2}^{b/2}(T - T_i)dy \quad (3\text{-}40)$$

This stress is a normal stress parallel to the x direction. At the free ends x = ± L/2 the normal stress is zero and significant deviations from the values given by Equation (3-39) exist in regions of the plate extending from x = −L/2 to x = −(L/2) + b and from x = (L/2) − b to x = L/2.

When the temperature distribution is not symmetric to the x axis and the thickness h of the plate varies with y, the thermal stress in regions distant from the free edges x = ±L/2 is given by

$$\sigma = -\alpha E[(T - T_i) - (T - T_i)_{av} - Cy] \quad (3\text{-}41)$$

where

$$(T - T_i)_{av} = (1/A)\int_{-b_l}^{b_u}(T - T_i)h\,dy \quad (3\text{-}42a)$$

$$C = (1/I)\int_{-b_l}^{b_u}(T - T_i)\,h\,y\,dy \quad (3\text{-}42b)$$

and A is the cross-sectional area of the plate. In these formulas y must be measured from the centroidal axis of the section; b_l and b_u are the distances of the lower and upper extreme fibers from the centroidal axis; and I is the moment of inertia of the section with respect to the neutral axis:

$$I = \int y^2 dA = \int_{-b_l}^{b_u} h\,y^2\,dy \quad (3\text{-}43)$$

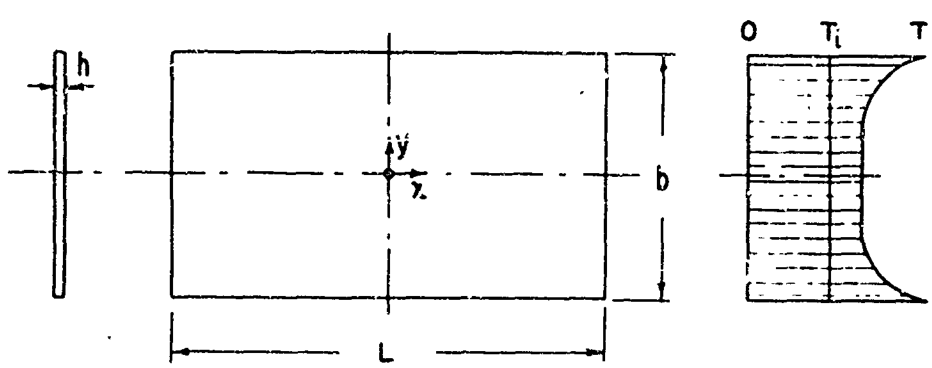

Figure 3-5. Assumed Temperature Distribution in Long Rectangular Flat Plate

3-6.3. Ring Frame. If a ring frame is heated uniformly, no thermal stresses arise in it provided it is made of a single material and is free to expand. If, however, the temperature of the outer surface of the ring exceeds by ΔT that of the inner surface, as indicated in Figure 3-6, the thermal change

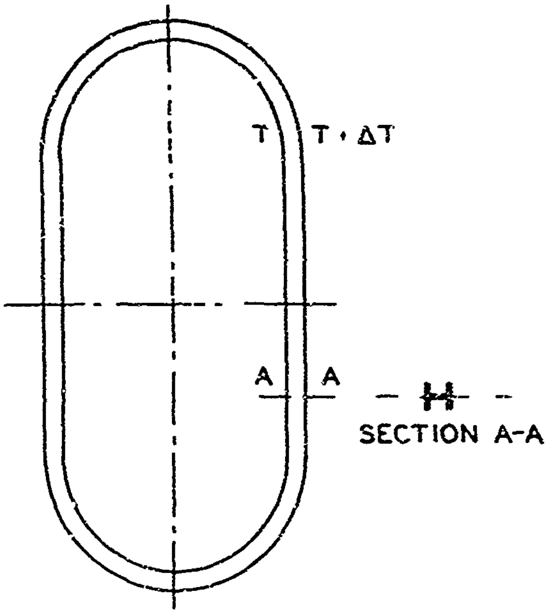

Figure 3-6. Ring Frame with Temperature Varying Linearly Through Depth

in the curvature is restrained by the continuity of the structure. When the temperature increase from the initial uniform temperature is a linear function of the depth of the section, but is uniform along the circumference and at the same time the ring frame has two axes of symmetry as shown in the figure, the maximum thermal bending stress is

$$\sigma_{b\,max} = \pm (1/2)\alpha E \Delta T \qquad (3\text{-}44)$$

In the derivation of this formula the additional assumptions were made that the cross section of the ring was doubly symmetric and that one axis of symmetry was in the plane of the ring.

When the temperature varies arbitrarily with y (through the section), but is constant around the circumference of the ring, the increase in temperature can be represented as the sum of an average increase, a linear variation with y, and an additional term $(T - T_l)_{ad}$ which varies with y. The first term causes no thermal stresses and the maximum stress caused by the second is given by Equation (3-44) with the stress distributed linearly across the section. The third term, $(T - T_l)_{ad}$, can be substituted in Equation (3-38) in order to obtain the additional thermal stress.

3-6.4. Circular Cylindrical Shell. The thin-walled circular cylindrical shell of Figure 3-7 is uniformly heated to a temperature exceeding by ΔT the uniform temperature of the ring frame. If the wall thickness of the shell is h and the cross-sectional area of the ring frame is A, the hoop compressive stress in the shell at the location of the ring is

$$\sigma_h = r\alpha E \Delta T \qquad (3\text{-}45)$$

and the maximum axial bending stress is

$$\sigma_b = 1.82 r\alpha E T \qquad (3\text{-}46)$$

provided that Poisson's ratio is 0.3. The value of the multiplying factor is

$$r = 1/[1 + (2/K)(Rh/A)] \qquad (3\text{-}47)$$

with

$$K = 1.29\sqrt{R/h} \qquad (3\text{-}48)$$

It can be seen from these equations that the value of r is always between 0 and 1. When the area of the ring is very small, r is very small, and the small amount of constraint provided by the ring does not give rise to large thermal stresses. When A is very large, r is not much smaller than unity, and the hoop compressive stress corresponds to almost perfect constraint.

3-7. THERMAL BUCKLING

3-7.1. Columns. The column does not know whether the compressive stress in it was caused by restrained thermal expansion or by an applied load. Consequently the customary buckling formulas can be used with confidence as soon as the compressive thermal stress is known. In particular, one may write for the critical stress of a simply supported column

$$\sigma_{cr} = \pi^2 E_t/(L/\rho)^2 \qquad (3\text{-}49)$$

where E_t is the tangent modulus (see Figure 3-3), L is the length of the column, and ρ is the radius of gyration of its cross section. The ratio L/ρ is known as the slenderness

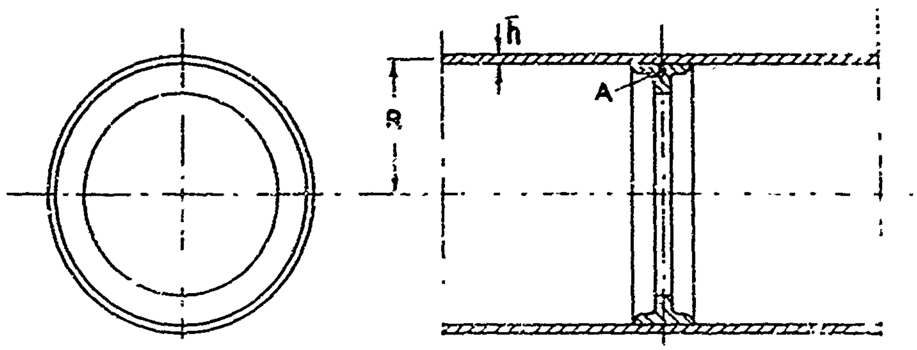

Figure 3-7. Ring-Reinforced Circular Cylindrical Shell

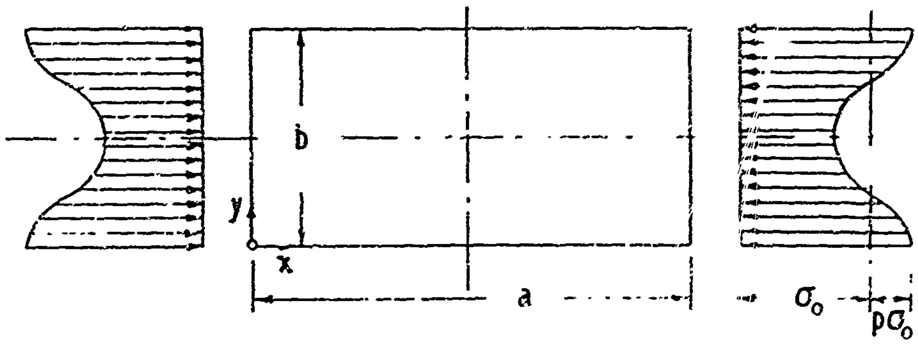

Figure 3-8. Simply Supported Plate Subjected to Thermal Stresses in One Direction

ratio. When the stress does not exceed the elastic limit of the material at the operating temperature, E_t becomes E.

3-7.2. Simply Supported Plates. In the case of plates the customary buckling formulas again are valid. Thus the buckling stress of a simply supported rectangular plate of constant thickness is (see Figure 3-8)

$$\sigma_{pl,o} = 3.6E(h/b)^2 \qquad (3\text{-}50)$$

where $\sigma_{pl,o}$ is a uniform compressive stress acting in the x direction.

The complication arising from the thermal nature of the stresses is that the distribution is usually not uniform. For a rigorous evaluation of the conditions of buckling, reference should then be made to the literature. When the distribution can be approximated by the formula

$$\sigma = \sigma_o[1 + p\cos(2\pi y/b)] \qquad (3\text{-}51)$$

buckling takes place at the following critical value of the average stress:

$$\sigma_{o,cr} = \sigma_{pl,cr}/[1 - (p/2)] \qquad (3\text{-}52)$$

The following examples will illustrate the meaning of this formula: When p = 1, from Equation (3-51) the stress at the two edges (y = o and y = b) is $2\sigma_o$, while in the middle (y = b/2) it is zero. Equation (3-52) yields in this case $\sigma_{o,cr} = 2\sigma_{pl,cr}$. Hence the shifting of the compression toward the supported edges and the consequent relief of the unsupported middle portion increases the critical value of the average stress by a factor of 2. On the other hand, p = −1 increases the compressive stress in the middle to $2\sigma_o$ and decreases it along the edges; the result is a decrease of the critical value of the average stress by a factor of 2/3.

3-7.3. Rectangular Plates with Free Edges. The control surfaces of missiles can often be considered as flat plates of constant or variable thickness with three edges completely free and the fourth constrained elastically (see Figure 3-9). If the stresses σ acting in the x-direction and caused by non-uniform rises in temperature are distributed as shown in the figure, the control surface is likely to buckle in a torsional mode when the stress intensity becomes sufficiently high.[11] The stress can be given as

$$\sigma = \sigma_o f(y) \qquad (3\text{-}53)$$

where σ_o is a constant while $f(y)$ defines the shape of the stress distribution curve. The stress is considered positive when it is compressive. Buckling takes place when

$$\sigma_o = GC/I \qquad (3\text{-}54)$$

where GC is the St. Venant torsional rigidity of the plate and

$$I = \int_{-b}^{b} hy^2 f(y)dy \qquad (3\text{-}55)$$

This integral can always be evaluated graphically or numerically without difficulty. When the plate thickness is constant,

$$GC_{h=const} = (2/3)h^3bG \qquad (3\text{-}56)$$

and G is the shear modulus of the material.

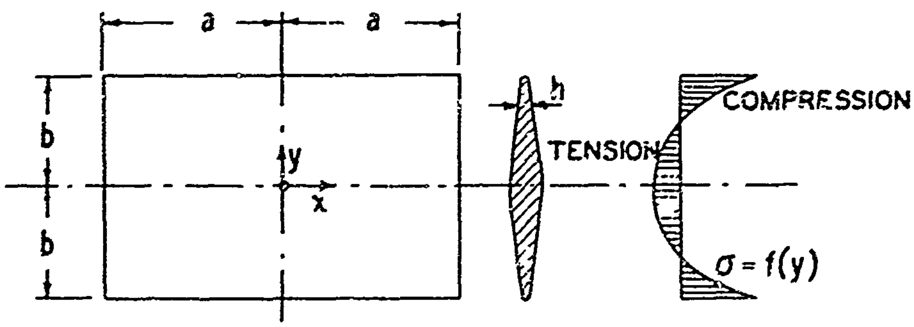

Figure 3-9. Assumed Stress Distribution in Rectangular Flat Plate with Free Edges

3-7.4. Thin Circular Cylindrical Shells. In order to simplify the problem of thin circular cylindrical shells one may consider separately three cases of temperature and thermal stress distribution. First, the temperature can vary through the wall thickness, but remain constant along the axis of the cylinder and around the circumference. In this case thermal buckling does not occur.

When the temperature is independent of the circumferential coordinate and is constant through the wall thickness but varies with the axial coordinate, the resulting hoop compression can, in principle, cause buckling. Investigations have shown,[11, 12] however, that under the practical conditions of ballistic missile design the danger of elastic buckling is very remote and that at most some localized inelastic dimpling may occur near restraining ring frames when the temperatures become very high.

The situation is different when the temperature varies only in the circumferential direction and remains constant through the wall thickness and with the axial coordinate.[16] A typical situation of this kind arises when a circular cylindrical shell is stiffened by means of a number of internally arranged stringers which are parallel to the axis of the cylinder. If aerodynamic heating raises the skin temperature very rapidly and the heat flow to the stringers is relatively slow, the stringers are subjected to tensile stresses and the shell wall to compressive thermal stresses in the axial direction. Under these stresses the shell wall buckles when the average intensity of the stress reaches approximately the critical value given in Equation (3-25).

3-8. CREEP EFFECTS

When the upper end of a vertically arranged tensile test specimen is attached to a rigid frame, and a weight is suspended from its lower end, the observer can measure an instantaneous elastic, and possibly a plastic elongation at the moment the load is applied. If the temperature of the specimen is sufficiently high, the instantaneous elongation is followed by further elongations which increase with time even though the load remains constant. These elongations are designated creep deformations.

The physical laws governing creep have not yet been explored sufficiently to explain completely the reasons for creep and to predict accurately the creep deformations under prescribed conditions of loading. Moreover, creep is influenced not only by the chemical composition of the material, but also by heat treatment and cold work. For this reason care should be taken in the application of creep formulas; they may be valid for a material before, but not after the manufacturing process.

Figure 3-10 presents the usual shape of the creep curve as obtained in a tensile test carried out under a constant load at constant temperature. The instantaneous deformation is followed by a period of creep at decreasing rates of strain, the so-called primary, or transient, stage of creep. After a while the time rate of strain becomes constant; the corresponding straight line in the diagram represents the secondary, or steady, stage of creep. Finally the creep rate increases again in the tertiary range until the specimen fails by fracture.

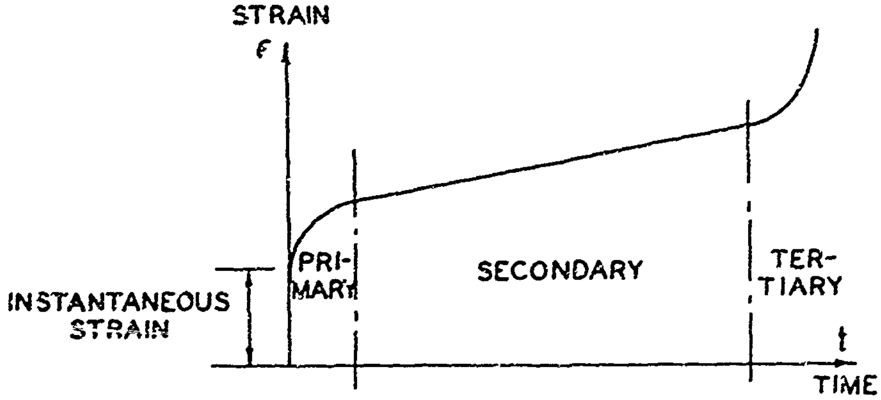

Figure 3-10. Tensile Creep Curve

3-8.1. Secondary Creep. The greatest amount of information is available today on secondary creep. For a given material the creep strain rate $d\epsilon/dt$ depends only on the stress and the temperature (t is time). On the basis of test results it appears possible to represent the creep rate as the product of a function $f(\sigma)$ of the stress σ and a function $F(T)$ of the temperature T:

$$\dot{\epsilon} = d\epsilon/dt = f(\sigma)F(T) \qquad (3\text{-}57)$$

The most generally accepted forms of $f(\sigma)$ are:

$$f_1(\sigma) = k_1(\sigma/\lambda)^n \qquad (3\text{-}58)$$

$$f_2(\sigma) = k_2 e^{C_2\sigma} \qquad (3\text{-}59)$$

$$f_3(\sigma) = k_3 \sinh C_3\sigma \qquad (3\text{-}60)$$

In these formulas k, λ, C and n are constants. Of these, k has the dimension in. per in. per unit time, the dimension of λ is that of a stress, the dimension of C is the reciprocal of a stress, and n is a number. Results of creep tests carried out with a material in a given condition of heat treatment and cold work indicate which one of the three forms is most suitable for an analytic representation. When the form is selected, the constants can be chosen to obtain the best possible fit of the empirical curve with the test points. The form of the function of the temperature is usually given as

$$F(T) = e^{H_u/T} \qquad (3\text{-}61)$$

where the constant Btu has the dimension of a temperature.

Although these formulas represent the results of constant-stress tests, they have been generally used under variable stress conditions also. They are more likely to give reliable results during loading than during unloading. The values of n in ballistic missile applications usually range between 3 and 15.

3-8.2. **Primary Creep.** During the primary stage of creep the creep rate changes with time. Results of tests carried out at constant stress and constant temperature can usually be represented as

$$\epsilon = k_4 (\sigma/\lambda)^n t^{1/p} \qquad (3\text{-}62)$$

However, this equation is unsuitable under conditions of variable stress. It can be brought into a form in which time does not appear explicitly; this is necessary because the presence of t in a formula representing creep strain leads to logical inconsistencies under variable stress conditions. Discussion of this can be found in the literature.[17] This assumption on which the transformation is based is that the creep rate at constant temperature is only a function of the instantaneous values of stress and strain but not of the history of loading. This assumption implies the existence of a mechnical equation of state. The recommended formulation is

$$\dot{\epsilon} = \frac{k_4{}^p}{p} \frac{(\sigma/\lambda)^{np}}{\epsilon^{p-1}} \qquad (3\text{-}63)$$

where the constants have the same values as in Equation (3-62).

As the value of p is usually between 2 and 3, the strain rate decreases with increasing strain, that is with increasing time. For this reason materials whose creep behavior can be expressed by Equation (3-63) are often referred to as strain-hardening materials.

Again, this creep law is more reliable in applications in which the load increases than in which it decreases.

3-8.3. **Stress Distribution Accompanying Creep.** The presence of creep changes the stress distribution in a structure from the one calculated from the linear theory of elasticity. The only exception to this statement is the stress distribution accompanying linear creep; this is characterized by $n = 1$ in Equations (3-58) and (3-62). Linear creep is occasionally observed with high polymers but not with metals.

Since the material, in general, is subject to elastic deformations even though creep deformations are also present, the stress distribution caused by a prescribed loading depends on the interaction between elasticity and creep, and it changes with time. At the moment when the loads are applied, the linear elastic distribution prevails. With increasing time this changes over into one that is governed by the creep law alone.

This final distribution is the same whether the creep phenomenon is represented by Equation (3-58) for secondary creep or Equation (3-62) for primary creep as long as the value of n is the same. Its analysis can be carried out with the aid of analogues and minimal principles derived in the literature.[18 to 22]

As an example, the results of the analysis of the stress distribution in a solid rectangular beam subjected to a constant bending moment may be presented. If h is the depth of the beam and y the distance measured from the neutral axis of the cross section, the distribution of the normal stresses (bending stresses) is characterized by the equation

$$\sigma/\sigma_{max} = (2y/h)^{1/n} \qquad (3\text{-}64)$$

where σ_{max} is the stress in the extreme fiber and n is the exponent in the creep law. The maximum stress in turn is given by the formula

$$\sigma_{max} = \frac{2n+1}{n} \frac{Mh}{6I} \qquad (3\text{-}65)$$

where M is the applied moment and I the moment of inertia of the cross section.

In the case of linear creep or linear elasticity

$$\sigma/\sigma_{max} = 2y/h \qquad (3\text{-}66)$$

$$\sigma_{max} = Mh/2I \qquad (3\text{-}67)$$

The stress distribution is linear only when creep deformations are absent, or when they are governed by a linear law as is the case with some high polymers. With metals the final stress distribution is non-linear since the creep law is non-linear.

The ratio R of the maximum stress in the presence of creep to that caused by purely elastic (linear) deformations depends only on the value of n:

$$R = \frac{\sigma_{max\ creep}}{\sigma_{max\ elast}} = \frac{2n+1}{3n} \qquad (3\text{-}68)$$

a few typical values follow:

n = 1	3	5	7	9	∞
R = 1	0.778	0.733	0.714	0.704	0.666

3-8.4. **Failure.** A tensile specimen subjected to a constant load fails by necking down and fracturing through the smallest cross section. When the material is metallographically stable and when its creep deformations follow the law of Equation (3-58), the critical time, that is the time elapsed between load application and fracture, can be given as[24]

$$t_{cr} = 1/n\dot{\epsilon}_o \qquad (3\text{-}69)$$

where $\dot{\epsilon}_o$ is the creep rate at the moment of load application

$$\dot{\epsilon}_o = k_5(\sigma/\lambda)^n \qquad (3\text{-}70)$$

with k_5 the experimentally determined constant at the proper temperature:

$$k_5 = k_1 e^{B/\theta T} \qquad (3\text{-}71)$$

When the material is metallographically unstable, empirical laws of the type

$$t_{cr}^m \dot{\epsilon}_o^q = \text{const.} \qquad (3\text{-}72)$$

are often derived from experiments. Much recent information on creep rupture can be found in Reference 24.

With a machine or mechanism which binds and becomes inoperative if a maximum permissible deformation is exceeded, the analyst must calculate the time at which the corresponding creep strain is reached. An approximate evaluation of this time is facilitated by isochronous stress-strain curves; they are a family of curves in the stress-strain plane with t = const. as the parameter. The creep behavior at different temperatures can be correlated by means of parameters discussed in Reference 24.

3-8.5. **Creep Buckling.** When a column (see Figure 3-11) is subjected to two equal and opposite compressive loads P, small bending moments M = Py_o arise in its cross sections because of the unavoidable deviations y_o of the center line of the column from the line of load application. The small lever arms y_o upon which the load P is acting can equally be caused by inaccuracies in the manufacture of the column and in its centering in the loading machine. Under these bending moments the small initial curvature of the column increases in consequence of bending creep. The increased curvature, in turn, leads to increased bending moments because of the increases in the lever arms y. As creep in metals is non-linear, the increases in curvature and deflection take place very slowly at the beginning, but they are accelerated in the later phases of the creep column test under constant load.

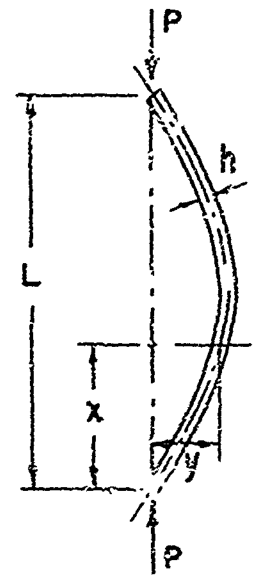

Figure 3-11. Deflected Column

Probably the most interesting fact about the buckling of a column in the presence of creep is that while the column is stable in its initial slightly curved state, it becomes unstable when the creep process increases its curvature sufficiently. This can be shown[21,25,26] with the aid of the stress-strain diagram of the material established at the temperature of the column test Figure (3-12).

At the moment of load application the deviation y is so small that virtually the same stress σ_o prevails throughout the cross section. But as the curvature increases in

consequence of the creep deformations, the convex side of the middle of the column will be under the action of a compressive stress σ_1 considerably smaller than the compressive stress σ_2 on the concave side of the middle of the column. When the bending moment is increased slightly, the stress on the concave side increases along the tangent to the curve. The slope of this tangent, namely tan α_2, is the tangent modulus E_t as discussed under paragraph 3-4.9. On the other hand, on the convex side the stress decreases along a line parallel to the initial straight portion of the stress-strain curve because only the elastic part of the deformations can be regained. The slope of the initial straight line, tan α_0, is Young's Modulus E.

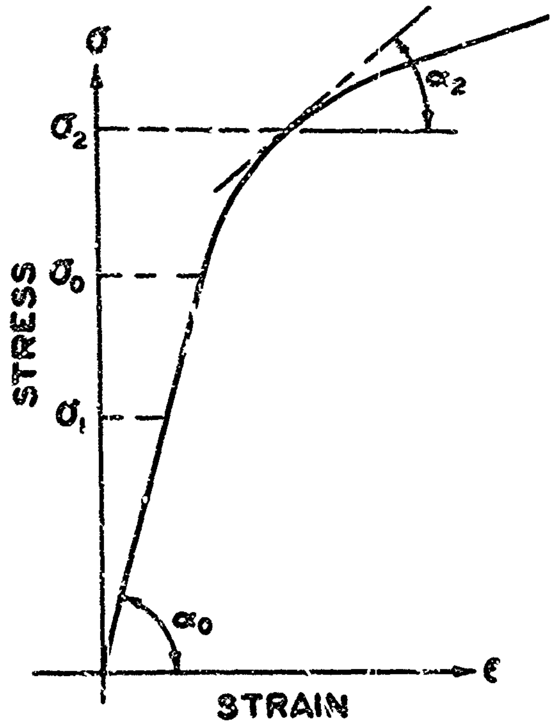

Figure 3-12. Stress-Strain Curve at Elevated Temperature

When the column is an idealized I section in which all the material is concentrated in two flanges a distance h apart, the effective flexural rigidity can be calculated from the values of E and E_t with the aid of the formula

$$(EI)_{eff} = I\,\frac{2EE_t}{E+E_t} \tag{3-73}$$

For other sections Equation (3-72) represents a satisfactory approximation.

The buckling load of the column is

$$P_{cr} = \frac{\pi^2(EI)_{eff}}{L^2} \tag{3-74}$$

and the critical stress can be given as

$$\sigma_{cr} = \frac{\pi^2 E_{eff}}{(L/\rho)^2} \tag{3-75}$$

where L is the length of the column, ρ, the radius of gyration of its cross section, and E_{eff} is simply

$$E_{eff} = (EI)_{eff}/I = 2EE_t/(E + E_t) \tag{3-76}$$

The value of E_{eff} decreases as the deflection of the column increases. Thus the critical load P_{cr} also decreases until, at the critical value of the deflection, it becomes equal to the applied load P. At that moment the column collapses.

Consequently, the critical deflection must be calculated from the instantaneous stress-strain curve of the material established at the proper temperature and after the proper time of exposure. The critical time is then simply the time necessary for the creep deformations to increase to the critical value. For the calculation of the critical time, data on creep are also needed. Since all this information is not yet available for the materials of construction, no closed formulas can be given for the critical time of columns.

Efforts have also been made to calculate the critical time of initially perfect columns. The results obtained, however, cannot be logically correlated to the practical buckling times observed in experiment.[27]

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Chapter 4

DESIGN CONSIDERATIONS

4-1. GENERAL COMMENTS

The design of a complete weapon system is a rather formidable task requiring the services of scientists and engineers in a variety of fields and a high degree of coordination of effort.[1] While attention in this chapter is focused on structural design considerations, it will be seen that these considerations are intimately connected with the guidance, control, propulsion and other aspects of the entire missile design.

The structural design problem can be visualized in a manner similar to closed loop systems for the guidance of aircraft and missiles. One starts, for example, with a study and establishment of the environmental conditions, goes on to a consideration of the mission of the missile and then to a set-up of the design criteria. At this stage, the choice of a propulsive system may be made. The structural analysis and design is undertaken, followed by laboratory tests of components or assemblies. The final stage involves the flight tests of the vehicle with instrumentation to check the environment assumed at the outset. Thus one arrives at the start of the loop and refinements are made by going through the loop as many times as necessary.

It may be well to distinguish between structural analysis and structural design. In structural analysis, the structure, loads and boundary conditions are given, and the analyst is required to find the stresses and deformations. In a design problem, however, a set of loads are to be transmitted over given distances, subject, for example, to a weight and stiffness specification. The designer must tentatively choose a material, structural configuration, and hardware cross-section which he feels will meet the specifications. The structure, under the given loads, is now checked against the requirements, and one is back to analysis once more

The accent in this chapter is on the design and analysis portion of the closed loop outlined above. Some of the most important aspects of the environment, mission, or design criteria of the missile are considered, one at a time, and their effects on the design are noted. While the treatment is qualitative in nature, the discussion will point up the manner in which the selection of materials, structural configuration, and cross-sectional areas are affected by the environment, mission and design criteria.

The radically different environments and missions of the many varieties of missiles precludes the possibility of establishing one set of design criteria. A particularly good example is the importance of the thermal environment, especially in the case of long range ballistic missiles. For these missiles, high speeds are obtained at high altitudes and a whole array of problems arise when the missile enters the lower, denser atmosphere at the end of its trajectory. These are usually called re-entry problems and involve all aspects of the structural design. Design criteria must be intelligently set up for each missile design from a careful study of all the information available on the environment, mission, requirements of rigidity, deformation, weight and balance, and others.

4-2. AERODYNAMIC INFLUENCE ON EXTERNAL CONFIGURATION

One of the major problems confronting designers of high speed aircraft and missiles is the role that aerodynamics plays in fixing the external configuration of the missile. From the structural designer's standpoint, aerodynamic considerations impose restrictions on the overall size and shape of the structure, and therefore on the capacity to transmit applied loads. If the restrictions are very severe, it may be necessary to seek better materials or more efficient structural configurations. A typical example is the wing

of a high speed aircraft or the control vanes of a missile. The wave drag is proportional to the square of the wing thickness, hence the aerodynamicist would like to have as thin a wing as possible. The ability of the structure within the wing to carry certain loads, however, increases approximately as the square of the wing thickness. In general, such cases of conflicting interests are settled by a compromise; this is almost always true in design problems. Should the aerodynamic requirement be very severe and have high priority, decreasing the wing thickness may lead to a prohibitively high weight for the wing design. In this event, the structural designer may have to study the possibility of a more efficient structural configuration, stronger alloys of conventional materials, or even the use of a new material.

The example cited has been over implified in order to illustrate the manner in which the interests of one group affect those of another. One could go further with the above example and show the effect that an extremely thin wing may have on other aspects of the design. The thin wing may introduce problems in connection with the housing of certain components. Generally speaking, changes in one design feature will affect many other aspects of the design; the importance of making intelligent compromises cannot be overemphasized in the success of an aircraft or missile design.

4-3. AERODYNAMIC HEATING

Probably the most serious problems arising from the aerodynamics of flight that affect the missile structural designer are those associated with aerodynamic heating. Depending on the speed, character of the boundary layer, and other factors, the heat input over crtical portions of the surface of a missile can attain values of the order of 10^6 Btu/ft^2/hr. The qualitative and quantitative aspects of this phenomenon are covered in another volume of this series.

The effects of aerodynamic heating on nose cone models are shown in Figures 4-1 and 4-2. These tests were run in the hypersonic tunnel at the Polytechnic Institute of Brooklyn. The model shown in Figure 4-1 was made of cast aluminum alloy and had a wall thickness of 0.5 inch. After 50 seconds of heating at a tunnel stagnation pressure of 100 psia and a stagnation temperature of 1995°R, a moderate amount of melting occurred at the stagnation region. Of interest is the symmetric manner in which the molten metal recrystallized after being carried downstream to the relatively cooler surface of the model. The model shown in Figure 4-2 was tested for 62 seconds. The model and test conditions were identical to those described above except that the stagnation temperature was 2030°R. The melted portion of the nose cone was observed to have disintegrated into relatively large particles.

Structural design for aerodynamic heating is undobutedly one of the major problem areas for the missile designer and analyst.[2] Under the influence of high heating rates, and the resulting high temperatures, materials which had performed satisfactorily in the past may be wholly inadequate. In the most critical regions (nose cone, control surfaces), several solutions may be considered by the designer to cope with the adverse effects of elevated temperatures on material properties and structural behavior of missile components. Even in regions subject to moderate conditions of thermal environment, the structural design problems are formidable. Structural design configurations for aerodynamic heating may be conveniently classified as (1) unprotected, (2) insulated, (3) coolcd, or a combination of these.

4-3.1. Unprotected Structures. Unprotected structures may be defined as structures that do not employ any fluid coolant or special insulators to deal with the thermal input from the boundary layer air. They present unusually difficult design problems inasmuch as the designer is concerned directly with material properties and structural behavior at elevated temperatures, aeroelastic, and propellant task problems. In addition, the designer must seriously consider transfer of heat to the interior of the missile because there may be temperature limitations on the proper functioning of electronic and other

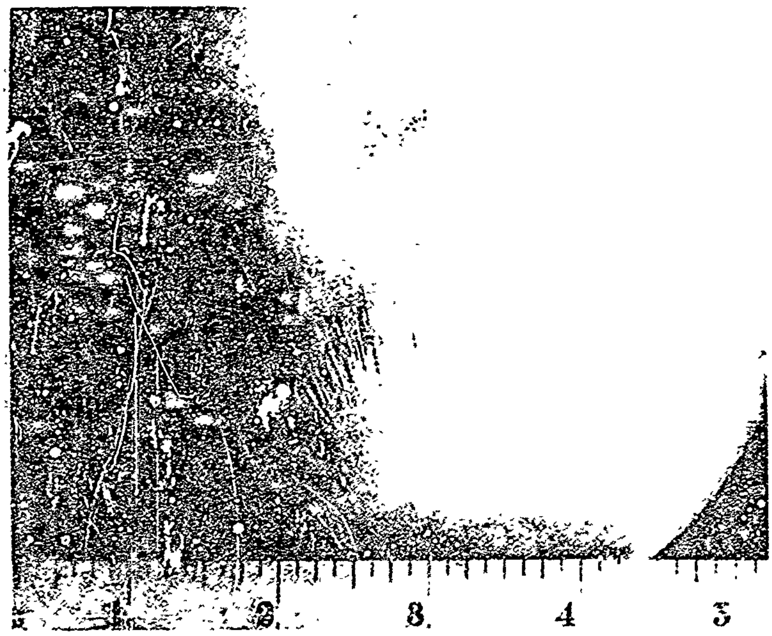

Figure 4-1. Effects of Aerodynamic Heating on Model Nose Cone, Stagnation Temperature 1995°R

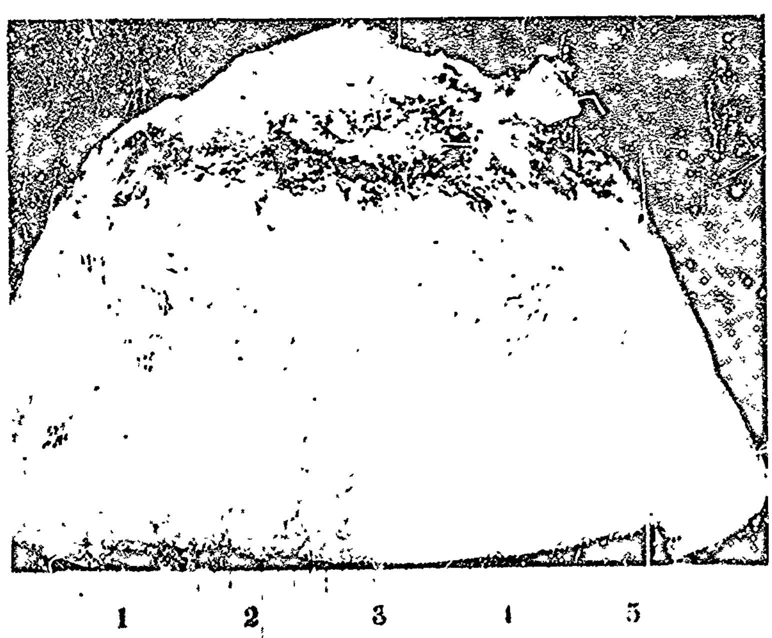

Figure 4-2. Effects of Aerodynamic Heating on Model Nose Cone, Stagnation Temperature 2030°R

equipment. The unprotected design should of course be compared with insulated and cooled designs in order to arrive at an optimum of configuration.

Unprotected structures have been classified [2] into three categories depending on the manner in which the effects of aerodynamic heating are handled: (a) heat-sustaining, (b) heat sink, and (c) thermal stress reducing types. On the heat sustaining type of structure the most complex problems arise from the effects of thermal expansion which cause structural growth, structural distortion, and thermal stresses. Structural growth, for example, gives rise to detail design problems in the attachment of internal equipment to the missile body and of surface control which compensates for growth of the structure. Structural distortions are unavoidable in the unprotected type of structure. Even if equilibrim temperatures are attained, temperatures are not uniform over the surface of the structure due to variations in heat transfer coefficients and local concentrations of material which serve as heat sinks. As mentioned in the early part of this chapter, variations in temperature from an average temperature give rise to thermal stresses and attention must be given to the possibility of stress concentrations or thermal buckling.

In the heat sink approach, extra material is added to the external surface of the missile. This addition of material has the effect of reducing temperatures, increasing material strength and reducing structural growth and distortions. The thermal stresses, however, may not be changed significantly. This solution to the aerodynamic heating problem is most efficient when adiabatic wall temperatures and flight times are such that additional material can be added to a light alloy structure. This is because specific heats of the light alloys are considerably higher than the specific heats of alloys of steel and titanium. For very short exposure times, the mechanical properties of this configuration are far superior to those shown in Chapter 2 for which exposure times may be one-half hour or more.

The stress reducing type of structure is an attempt to evolve a structural configuration that minimizes the thermal stresses caused by uneven distribution of material and variations in the heat transfer coefficient. For wing-like structures, trussed and corrugated webs and ribs have been investigated. Consideration has also been given to sandwich construction of the surface to surface and individual surface types, and the use of shallow, closely spaced stringers. In each of the configurations mentioned the basic idea is to avoid concentrations of material which give rise to temperature differences, and hence thermal stresses, in a structural component. The closely spaced shallow stringer config- them down into a number of smaller heat sinks. The shallow depth keeps the average stringer temperature close to the skin temperature, thus reducing the thermal stresses. Introducing a flexible connection or support between components avoids a build up of internal stresses due to constraints.

4-3.2. Insulated Structures. The insulated structure introduces the concept of one structure to absorb the primary loads and a second structure to cope with the thermal input. Consider, for example, a conventional structural configuration of a material which may have only fair high temperature characteristics. A coating of insulating material may be applied to the surface of the structure to keep the temperature of the load carrying structure at reasonably low levels. This composite type of structure, however, requires either a reinforced plastic or refractory for the insulating material in order to compare favorably with the double wall solution. Moreover, the insulating material has stringent mechanical and thermal requirements. It should be erosion and heat resistant, and it must possess good adhesive characteristics. Although some ceramics and refractories appear to have some of these desirable properties, it seems that the necessary requirements are too severe to be met by a single material. Many refractory materials are brittle and have low thermal shock resistance. Although ceramics in the oxide family have a lower thermal conductivity than those in the silicide, boride or carbide groups, a 0.10 inch coating of aluminum oxide would have little insulating value in

regions of moderate to high thermal inputs. Developmental work in this direction is nevertheless worthwhile because of the simplicity and low cost possibilities

A more promising and feasible insulated structure is a double wall construction. The outer shell is fabricated from a heat resistant material and is relatively thin and light. The panels of the outer shell essentially carry only the local normal loads, and therefore need be designed to withstand these loads in only at the elevated temperature. The inner wall or shell is the primary load carrying structure and may be conventional in design and material. The reason for this is that the inner structure will not be subject to high thermal inputs and the temperature rise is kept low enough that material properties are not significantly affected. Interposed between the walls is a suitable light and effective insulator. For relatively close spacing between the walls, air alone might serve effectively as the insulating medium. If the outer panels are designed in such a manner that each individual panel can expand or contract freely, it offers the advantage of complete absence of thermal stresses. The outer panels can be allowed to float by attaching them to the primary structure with thin strips or clips which allow expansion of the panel in its own plane. In addition to the advantage of complete absence of thermal stresses, other features of this design include its feasibility from the standpoint of developmental time, heat protection for internal equipment, and freedom from panel flutter.

Some of the most interesting insulated structure solutions to the aerodynamic heating problem have been proposed for missiles of long range re-entering the atmosphere at high speeds. One of these solutions is the non-melting heat sink. In this application, the primary or load-carrying structure is covered with an engineering material in sufficient quantity that it can absorb the thermal input without melting. Insulation is interposed between the outer heat-absorbing material and the inner load-carrying structure in order to keep the load-carrying structure relatively cool. It is to be noted that this heat sink is quite different from the heat sink unprotected type structure. In this latter case, the mass of the unprotected primary structure was simply increased in order to enlarge its heat absorption capacity. The heat sink approach has received a great deal of attention because of its simplicity. Copper appears to be a suitable cover material for heat sink purposes primarily because of its excellent thermal diffusivity. The poor high temperature mechanical characteristics of copper have not affected its use for this purpose.

Should the thermal input conditions be exceptionally severe, the designer may still utilize the heat sink approach, except that the heat sink material is allowed to melt. The molten material is then ablated or carried away by the windstream. This approach has been used successfully in several re-entry bodies. One of the major problems in the application of this solution is the instability of the ablation process. Any irregularity or pit in the surface may increase the heat transfer locally, melting material at a greater rate and precipitating a very rapid and wholesale ablation of the material in the entire area.

Sublimation cooling is another interesting and promising approach to the aerodynamic heating problem for very high thermal inputs in which the thermal shield material itself is sublimated at the surface. The sublimation cooling solution is here classified as an insulated structure inasmuch as there is a primary structure which is being protected by the outer cover of material which is sublimating In common with the nonmelting and melting heat sink discussed in the preceding paragraphs, the sublimation cooling technique is basically one in which a thermal shield protects a primary load-carrying structure. Materials which show promise as outer covers, are poor thermal conductors, and it appears that there is no necessity to interpose insulation between the primary structure and the outer cover.

The principal beneficial effects of sublimation cooling are in the reduction of the heat transfer coefficient due to introduction of

fluid material into the boundary layer, and in the large heat required for sublimation. It is also reported that uneven heating or local hot spots are absent. Moreover there is a built-in stability in the system; local high heating increases the sublimation rate at that location, thus affecting a reduction in heat transfer. A possible defect in this approach exists in connection with the use of low heat of sublimation materials which may blow so much material that the characteristics of the boundary layer could be altered. This could lead to large scale turbulence phenomena with attendant higher heating rates. It has also been reported that burning accompanying sublimation does not seem to be a serious matter.

Another method for protecting the structure consists of the application of surface coatings which maximize the radiation of heat. Materials such as cerium having high radiant emissivity characteristics may be utilized for this purpose.

4-3.3. **Cooled Structures.** Under certain conditions of the missile mission and expected thermal environment, a cooled structure may be necessary or desirable. It appears desirable to consider direct cooling of the primary structure for small missiles where space is not available for insulation or for very short flight times.

Cooling of the primary structure may be accomplished by an internal cooling system or by porous cooling. In the internal cooling system, a fluid is circulated through the structure to effect a transfer of heat from the structural material to the fluid. The coolant may be circulated in channels which are an integral part of the structure or otherwise in separate ducts which are in intimate contact with the structure. For long range missiles or where there is a re-entry problem, it appears that the high heat capacity of the low atomic number metals is an attractive property. These metals would be used in the molten state. Some of the undesirdable characteristics mentioned have been their poor thermal conductivities and difficulties of fabrication and handling.

In a porous cooling solution, the structural material must be porous in order to allow the passage of the fluid through its thickness. The structural material may be manufactured by powder metallurgy techniques or constructed by compressing layers of screens to the desired thickness. The fluid may be either a gas or a liquid, but probably a liquid coolant would be preferable in order to take advantage of the large heat absorption associated with the phase change and the decreased container requirements. There are a number of interesting features associated with this method of cooling a structure. It appears that blowing through the boundary layer reduces the magnitude of the heat transfer coefficient which is, of course, desirable. There is also a built-in stability in the system inasmuch as a local hot spot will evaporate more fluid at that point. Under these conditions, more heat is absorbed at the location in question and the net effect is a tendency to make the thermal inputs more uniform over the surface of the missile. Structures employing a combination of cooling and insulation techniques are also feasible.

4-4. DESIGN FOR APPLIED LOADS

Design for the transmission of the applied loads requires a careful study of critical loading conditions, material properties and structural configurations. In order to enable the structural designer to predict required hardware cross-sectional areas, there must also be available a rational set of structural design criteria. Where there exists a long history of experience with a particular type of structure and its environment, adequate design criteria have been established. Unfortunately, this is not the case in the missile field. The experience accumulated thus far is relatively limited, and a great deal of complication has been introduced by the dependence of the design criteria on both temperature and time.

While details of the considerations involved in obtaining design loads, factors of safety, and allowable stresses and deformations will not be discussed, the more carefully these items are selected, the more effi-

cient will the design be from the structural standpoint.

4-4.1. Types of Loads. Stress analysts frequently speak of *primary* and *secondary* loads. Primary loads are the major loads that must be resisted and transmitted by the structure. Consider, for example, that under a particular flight condition, certain loads are acting on the missile structure. If we cut off the tail portion and examine the equilibrium of this portion as a free rigid body, then three forces and three moments will be required at the section of the cut in order to maintain equilibrium. These may be considered as the primary loads being transmitted at the section.

Secondary loads are self-equilibrating. They are a consequence of structural deformations rather than externally applied loads. Thermal stresses are an example of secondary stresses. Although secondary stresses can be disregarded in some branches of engineering, they cannot be overlooked in missile structural design.

The principal loads on the missile structure are aerodynamic, inertial, and thermal. There are also important loads which are imposed on the structure that do not fall precisely in the above categories, but which are nevertheless important in determining the structural design. These include loads induced by the propulsive system, those induced by internal pressurization, fuel slosh, vibratory loads, and loads encountered in transportation, handling and launching.

4-4.2 Limit and Ultimate Loads; Factor of Safety. The loads that are actually anticipated during the lifetime of a missile are called *limit loads.* These loads may be calculated by specialists in various fields and the information made available to the structural analyst and designer. It is possible that data in the form of tables and curves can be made available to the designer for preliminary design purposes. Limit loads are multiplied by a *factor of safety* to obtain *ultimate loads.* In the design of aircraft, factors of safety are specified; [3] these can be applied to missiles until such time as separate design criteria are established for missiles or if the procuring agency does not specify safety factors in a specific missile development. Factors of safety are applied to limit loads to take into account uncertainties in connection with the establishment of the limit loads, properties of materials, and methods of stress analysis.

4-4.3. Thin-walled Circular Cylinder Structural Design. Assume that the designer has tentatively chosen a thin-walled circular cylinder as a structural configuration; this choice based upon his experience and information available on the expected environment and mission of the vehicle. It may be convenient to consider that aside from the nose cone and control vanes, the circular cylinder comprises the entire body of the missile. One can now confine himself to the applied loads, and inquire into the efficiency of the structural configuration, and into the effect that the magnitude of the loads and design criteria have on material selection and required cross-sectional areas.

Figure 4-3 is a photograph of the tank section of the Jupiter missile illustrative of the thin-walled circular cylinder as a structural configuration. Note the extremely large ratio of radius to wall thickness and the circumferential reinforcement rings. The photograph in Figure 4-4 shows the top section of the Redstone being inspected prior to movement to the assembly area where internal equipment is installed. In the background is the nose cone in which the warhead is installed.

Treating the circular cylinder as a beam, the cylinder is required to transmit three force and three moment components at any cross-section. For simplicity, the two force components in the plane of the cross-section can be obtained into a single shear force, and the two moment components in the plane of the cross-section into a single bending moment. The remaining force component is an *axial* force, the remaining moment component is called the *torque.*

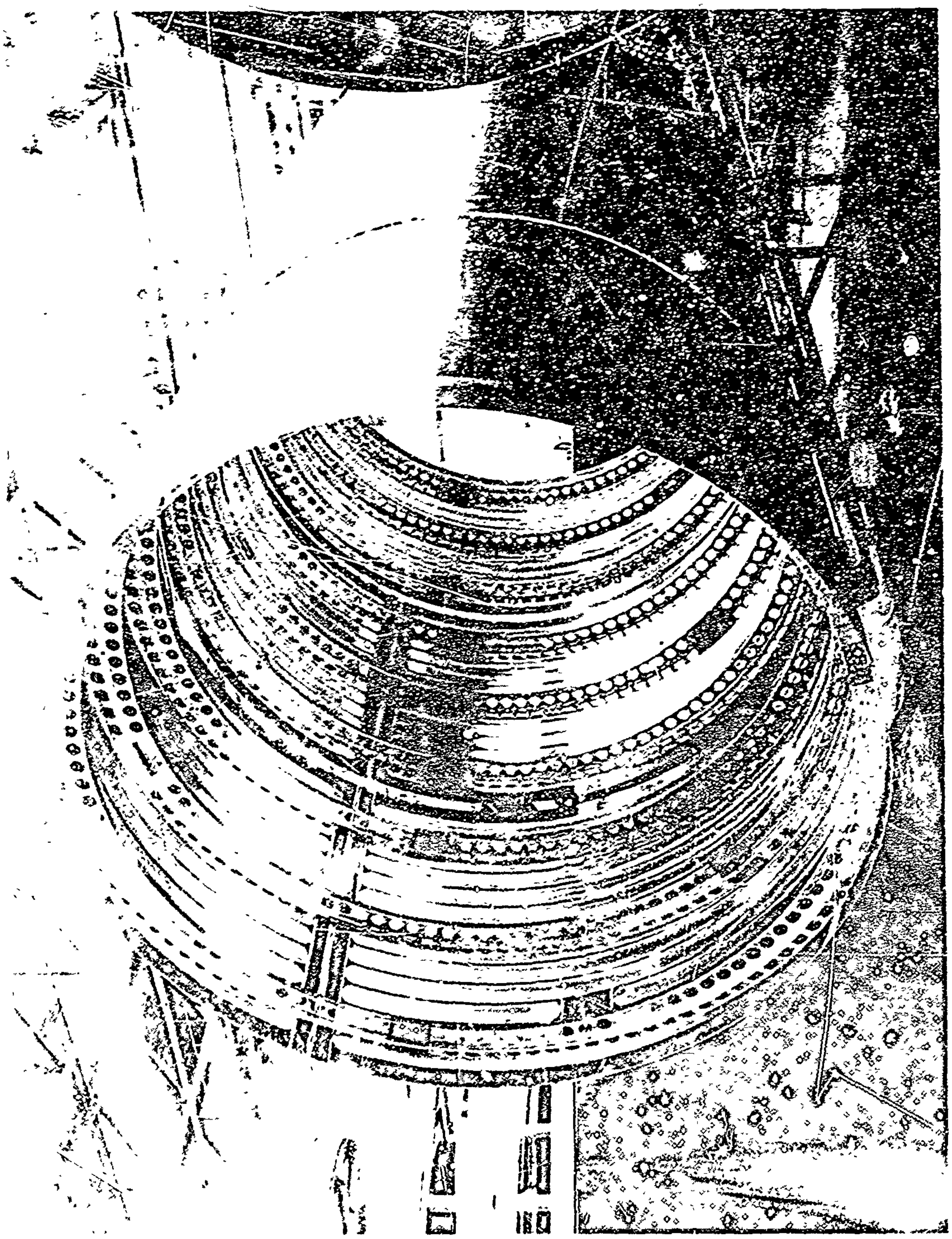

Figure 4-3. Tank Section, Jupiter Missile

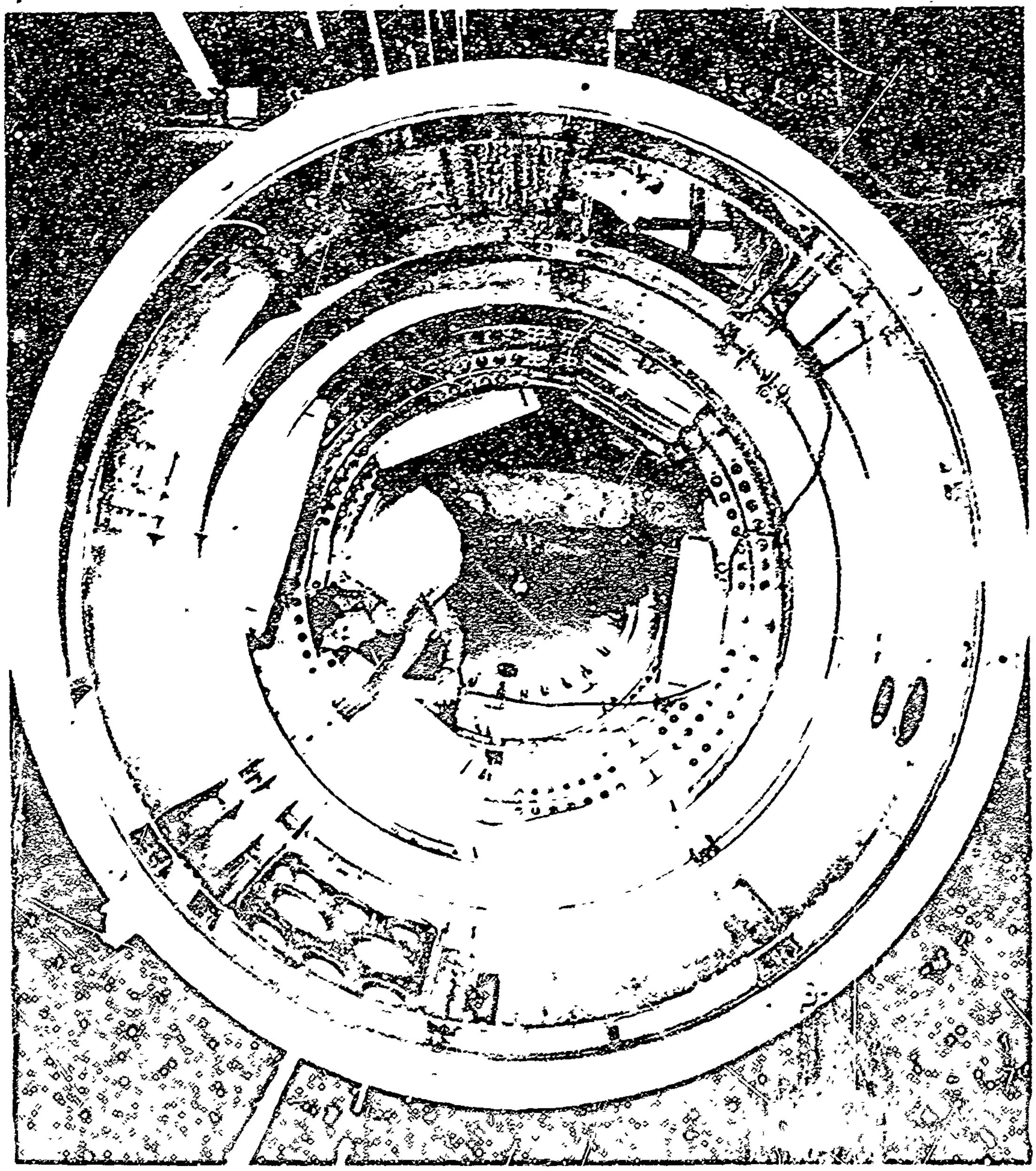

Figure 4-4. Top Section, Redstone Missile

Taking the shear force in the plane of the cross-section passing through the center of the circle, this force is transmitted by means of shear stresses which are symmetrically disposed about the line of action of the resultant force. The variation is sinusoidal; varying from zero at the point of intersection of the line of action of the force and the cylinder, to a maximum value at 90 degrees, and back to zero at 180 degrees. The maximum shear stress is twice the average value obtained by dividing the force by the cross-sectional area. The axial force gives rise to a uniform normal stress distribution whose value is given by the axial force divided by the cross-sectional area.

Turning now to the moments which are to be transmitted, it can be shown that the thin-walled circular cylinder is about one and one-half as efficient as a solid rectangular section. Comparing these cross-sections on the basis of equal areas and equal maximum normal stresses, the criterion is simply the *section modulus*, which is the ratio of the moment of inertia to the distance from the neutral axis to the extreme fibers. The relative efficiencies of the I-beam, circular cylinder and solid rectangular section are 1, ½, and ⅓ respectively. The thin-walled circular cylinder has a moment of inertia which is constant about any diameter. Thus this shape can absorb a bending moment acting in any direction with equal facility.

The remaining moment component, the torque, is most efficiently transmitted by the thin-walled circular section. In a circular section the *twisting rigidity* (torque per unit angle of twist) is proportional to its polar moment of inertia. A tubular cross-section has a higher polar moment than a solid circular section of the same area. The torque is resisted by a uniform distribution of shear stresses, and the moments by a distribution of normal stresses given by the well-known MC/I formula.

It is to be noted that considerations of buckling, concentrated loads, edge effects and others have been disregarded.

Normal pressures acting over the surface of the cylindrical shell are conveniently resisted by hoop or membrane stresses. Curved two-dimensional elements are naturally efficient in this respect. A flat two-dimensional element, or plate, can resist normal pressures only by means of its bending resistance. This generally gives rise to fairly high stresses. On the other hand, curved two-dimensional elements are able to supply tension force components opposite to the direction of the pressure vector and therefore are used wherever possible to withstand high normal pressures.

4-5. STRUCTURAL FAILURE

In analyzing the integrity of a design, the stress analyst considers yield and ultimate strength, stiffness, fatigue strength, and buckling. Assume that the designer has tentatively chosen a structural configuration and the material or materials of construction. Also assume that tentative cross-sectional areas have been established. The primary object then is to study the manner in which the design criteria affect the cross-sectional area. At a very early stage in the design a tentative structure must be *laid* out, or it would not be possible to calculate preliminary aerodynamic loads, thermal inputs, thermal stresses, inertia loads and other quantities.

4-5.1. Yield and Ultimate Strength. The criteria established in analyzing the yield and ultimate strength of a structure are essentially the following. At limit load, the structure should not suffer more than a certain, maximum permanent deformation. A 0.2% permanent set is usually specified. The integrity of the structure should be maintained, i.e., no collapse or failure up to the ultimate load. This means that the stress analyst, in checking the yield strength of the structure, must perform stress calculations for the limit loads under a variety of environmental conditions and ascertain that the stresses nowhere exceed the yield stress of the material of construction. In order to simplify the calculations, the analyst generally investigates the most critical conditions and the most critical portions of the structure. If the maximum stresses are below the yield stress of the material, then the structure has

sufficient yield strength. To check the ultimate strength, the analyst multiplies the limit loads by a factor of safety and goes through the stress calculation. The maximum stresses must everywhere be less than the ultimate stress of the material.

Should the yield stress of the material be exceeded under limit loads, or the ultimate stress exceeded under ultimate loads, then the structure is obviously too weak and the design must be strengthened. One of the most straight-forward measures that can be taken by the designer is to increase the cross-sectional area sufficiently that the maximum stresses are reduced below the yield or ultimate strength criterion, depending upon which one is critical. In the case of the thin-walled circular cylinder, the designer may increase the wall thickness. This solution, of course, adds weight to the structure which may well be intolerable. The designer may then consider another alloy of the same material with higher yield and ultimate strengths. This may give a satisfactory solution, provided that no trouble is anticipated due to increased cost, manufacturing difficulties, the possibility that the material is too brittle, and other considerations. The designer may also investigate the desirability of altering the structural configuration or the use of another material.

4-5.2. STRUCTURAL STIFFNESS

A measure of the stiffness of a structure is the product of Young's Modulus of the material and the geometry of the cross-section, namely the moment of inertia. This product, EI, is called the *bending rigidity*. *Torsional rigidity*, GC, is also the product of a material elastic constant and the cross-section. G is the Modulus of Rigidity and C is a property only of the cross-section. For solid or tubular cross-sections, C is the polar moment of inertia of the section. The stiffness of a structural element is also affected by the magnitude and type of load to which it is subjected. Compressive loads tend to decrease the effective stiffness of a structural element, whereas tensile loads have the opposite effect.

Strictly speaking, the stiffness criterion is not directly related to the strength of a structure. A stiffness criterion on a structure or structural component may be established because too much flexibility of the structure may interfere with the proper operation of the control system, or may seriously affect the stability and control characteristics of the missile. Consider the design of control linkage systems and their supports. It is obvious that if the supports were attached to a very flexible structure, the supports would fall out of line under the applied loads and could render the control system inoperable. The same would be true if the control shaft itself were to deflect transversely between supports. To insure the proper operation of such control systems, the design is governed by a stiffness criterion.

A number of other examples may be cited. It is important to consider the relative deformation between the sensing instrument, say in a stabilization system, and the control surface. If too much twisting or bending of the structure occurs between the location of the sensing equipment and the control surface, the desired stabilization may not be achieved. It is also possible to adversely affect the stability derivatives of a missile if there is excessive deformation in bending, or control effectiveness may be impaired if a control surface does not have sufficient torsional stiffness. The important point that must be borne in mind is that rigidity requirements are important considerations in the design of a missile and some of its components.

There are instances in which a rigidity requirement is imposed in order to safeguard the integrity of a structure. Under some flight conditions, the aerodynamic forces may vary in resonance with the natural frequency of the structure which in turn is a function of the structures stiffness and mass distribution. This resonance can lead to catastrophic failure. Interactions between aerodynamic, elastic, and inertia forces are called *aeroelastic* effects. The speed at which these forces resonate is called the *flutter speed* and the phenomenon is called *flutter*. The components of a missile that are most apt to

flutter are the control surfaces. Individual panels of a structure may also flutter under the proper conditions.

The above considerations are very important and the aeroelastician must calculate the flutter speed to insure that it is well above the speeds expected to be encountered. The analytical computations cannot be performed until the design has progressed sufficiently to enable the analyst to determine the elastic resistance and mass distribution of the structure. Aerodynamic and structural damping are usually included in the analytical formulation of the problem. Once the missile has been constructed, an experimental check can be made of the structural stiffness by statically loading the member or missile and observing the deflections and slopes, or by forced vibration techniques by which the stiffness can be deduced from the amplitude and frequency response of the structure.

It may also be worthwhile to mention that liquid sloshing and liquid impact against bulkheads at thrust cut-off may affect rigidity requirements of certain structural components.

4-5.3. Fatigue. The design of a structure for vibratory loads is a source of major concern for all missile structural designers. A vibratory load on a structural element immediately gives rise to the possibility of failure by fatigue. For flight vehicles that are designed to last (in principle) an indifinitely long time, the possibility of fatigue failure is important and always investigated. For ballistic missiles, however, the relatively short life duration usually precludes the possibility of fatigue failure. Fatigue failure should be investigated in a missile design, but it is generally not important as a design consideration.

An example under which a ballistic missile may be subject to fatigue failure can be cited. A ballistic missile is erected on its launching pad in such a manner that the stand load is introduced at the tips of the fins. The missile may remain in this position for a relatively long time. At a particular speed, lateral winds passing over the fins may cause the fins to resonate, introducing additional oscillating loads on the fin support points and thereby creating the possibility of fatigue failure.

The problems associated with vibratory loads are more in the nature of their effect on the performance of the missile rather than with the strength or integrity of the structural component. A vibrating panel could conceivably affect the aerodynamic behavior of the missile. At times the problem is associated with the proper functioning of electronic equipment or with acoustical transmission. There are also other problems created by the vibratory loads, some of which may be of greater concern to some designers than those mentioned above. Having noted that fatigue is not the important consideration, but rather one of interaction with other phases of the missile design, the question raised is in connection with the steps that the structural designer can take to alleviate the undesirable conditions.

One obvious method of approach is to eliminate the vibratory forces at their source. This cannot always be accomplished. However, if the source of the virbatory loads can be traced to the engine, or if vibrations occur in transit and storage, a shock mounting technique may be effective. In other cases, the designer must seek to increase the stiffness of the structure or portion of the structure which is involved. Inasmuch as the structural stiffness is proportional to the product of Young's Modulus (or the modulus of rigidity) and the geometry of the cross-section, the designer can either choose a new material with a higher Young's Modulus, or he can change the cross-section. Changing the cross-section to increase the stiffness can be achieved by altering the configuration without increasing the area, or by increasing the area.

4-5.4. Buckling. Shear stresses and normal compressive stresses acting on thin-walled elements give rise to the possibility of buckling. Fortunately, buckling of a curved two-dimensional thin-wall element requires a stress which is much higher than the buckling stress of a flat element. The buckling stress of a square flat plate simply supported

on all four sides and uniformly compressed in one direction can easily be calculated for elastic buckling. If the plate were to be rolled into a cylinder, the critical stress due to uniform axial compression would be about 200 times higher based on an original square plate of 12 inch sides and a thickness of 0.032 inch.

Failure by buckling is at times the decisive criterion in the selection of a material of construction and in the determination of cross-section areas. As an example of how buckling considerations can affect the choice of a material, consider a flat plate subject to edge compression in one direction. The buckling stress of the plate varies with the square of the thickness. A comparison of the buckling stress is desired between two plates; one of aluminum and one of steel. The plates are to have the same length and width, and the same weight. Since aluminum is about ⅓ lighter than steel, the thickness of the aluminum plate can be three times thicker for the same weight. Inasmuch as the buckling stress varies with the square of the thickness, the aluminum plate will have a buckling stress that is approximately nine times that of the comparable steel plate. This is one of the strong reasons that aluminum alloys are selected over steels in the design of most aircraft and missiles that do not operate at temperature in excess of say 400°F.

If buckling of a structural component results in the collapse or failure of the entire structure, it is clear that the buckling stress of the structural component would be the maximum allowable stress. In such an event, the structural designer must increase the cross-sectional area of the component until the value of the stress falls below the maximum allowable. It might be proper to point out, however, that buckling of a structural component does not necessarily correspond to failure of the entire structure. Consider, for example, a structure composed of a single bar in compression. The buckling load and maximum load or such a one-dimensional element are identical for all practical purposes. Buckling would correspond to failure of both the element and the structure. This is not the case, however, if the bar in question is but one bar of a redundant framework, because the other bars in the framework can absorb additional loads. A flat or curved two-dimensional element in edgewise compression in one direction will buckle at a relatively low load, especially if the thickness is small. This does not correspond to the maximum load or failure if the unloaded sides are supported. The material near the supports is constrained against transverse deflection and can absorb loads beyond those corresponding to the buckling load.

4-6. TIME AND TEMPERATURE DEPENDENCE OF DESIGN CRITERIA

Most of the mechanical design criteria previously discussed are applicable not only to missiles but also to other types of flight vehicles and to many types of ground based structures. What problems in this area then are peculiar to the design of missiles? The answer is that if one again goes back to considerations of aerodynamic heating and finds that the thermal inputs are significant, then design criteria established in connection with the strength and stiffness requirements must be re-examined. Reference 4 contains a more detailed discussion of the strength, stiffness and fatigue requirements for both cold and elevated temperature structures and proposes revised design criteria for structures at elevated temperatures.

Briefly, the complicating factor is the dependence of the design criteria, and even the stress analysis itself, on temperature and time parameters. Investigation of the yield strength of a heated structure may serve as an illustration. At room temperature, the yield strength of a structure, in the simplest case, is a matter of keeping the stresses below the yield stress of the material used in the construction of the structure. Room temperature mechanical data of this sort is abundant and easily available. At elevated temperatures, however, the mechanical properties of the material may undergo significant changes. Inasmuch as these data are also available, it would seem logical to use the

yield stress at an appropriate design temperature as a yield strength criterion. While this procedure may appear logical and expedient, the time parameter cannot be ignored.

A ballistic missile may be subject to significant thermal inputs and temperatures for relatively short time periods, i.e., of the order of several minutes. The temperature-time history is generally transient during this period. Even if a design temperature could be established for this period, the elevated temperature mechanical data at the same temperature may not be strictly applicable because these data are usually obtained after the specimens have been soaked at the test temperature for ½ hour or more. It is well known that the elevated temperature properties of a material are affected by the time element as well as by other time dependent factors such as the strain rate. Mechanical data for very short times are relatively scarce. Hence, a missile designer may have no recourse but to use the data that is available with the risk of being over-conservative.

A final consideration is the possibility of the occurence of creep of the structural material. The yield strength criterion in this case could be taken as the stress required to produce a 0.2% creep-set at a given temperature. The value of this stress is also a function of the time; the longer the time required to produce the 0.2% creep-set, the higher the allowable stress value (see Figure 2-12). For the short-time requirements at load and temperature for ballistic missiles, however, it appears that creep is not a major design problem. Nevertheless, there are some designers who feel that creep may become a major design problem. For example, a missile compartment must maintain atmospheric pressure at high altitudes because some of the components within the compartment would not function at lower pressures. The skin of this compartment is heated to a high temperature during the ascending phase of the flight due to aerodynamic heating and cools slowly while coasting outside the denser atmosphere. Because the differential pressure will cause high hoop stresses in the hot skin of the compartment, significant creeping can occur in a period of time corresponding to a few minutes.

Similar remarks can be made in connection with temperature-time-load effects on other structural strength requirements. The immediate objective of the preceding discussion was to show the significant effects that the thermal environment had in the ultimate structural design criteria that were established.

It is important to point out that the determination of the loads on a missile, and the establishment of the associated design criteria are not confined to design conditions which occur after the missile leaves the launching platform. The designer must also obtain information on the loads which may be imposed on the missile from the time of fabrication to the time of launching and establish appropriate design criteria. Pre-flight loads are important and not to be overlooked. They may be critical in the design of a structural detail or even in a major portion of the structure. Pre-flight loads may have to be investigated during production, handling, transportation, or while poised in the launcher. A particularly interesting example is that of a missile of thin-walled design which requires positive internal pressure to stabilize the skin against buckling, even while it is in production!

4-7. MISCELLANEOUS FACTORS AFFECTING STRUCTURAL DESIGN

A number of miscellaneous considerations affect in one way or another the final missile structural design. The considerations should not be assumed secondary or unimportant solely on the basis of space allocation in this discussion. Maintenance considerations, for example, are not separately discussed, yet a poor design would result if the designer neglected to make provision for replacement, repair, adjustment, access, etc., of various components of the missile. Indeed, some designers can point to one or more of the items

to be mentioned as the consideration that pervaded an entire design. Most of the following factors entering and influencing the missile structural design are not peculiar to the missile field, but rather to the general field of design of flight structures. This is the principal reason that little more than a listing of the various considerations is presented.

4-7.1. Time Schedule. As a first item, the effect of a time schedule is taken as a possible factor influencing a missile structural design. This factor is purposely taken at the outset because one normally might not conceive of the possibility of a development, production or delivery schedule profoundly affecting the final product. It is nevertheless true that if a strict deadline faces a missile designer, the final design may be quite different from the design envisaged had more time been available. One might imagine that a missile designer is intrigued by the attractive properties possessed by a particular material and seriously contemplates its use in the design. One or two important physical properties of this material may be unknown or uncertain, and the designer feels that an investigation of these properties is warranted before authorizing its use in the design. In the face of a tight time schedule and the uncertain length of time required to establish the acceptability of the new material, he may be forced to stand by a material with which he has already had experience, but which might have effectively been replaced by the new material.

4-7.2. Cost Considerations. In the structural design of the primary or secondary structure, the designer frequently has several possible solutions to the problem on hand. The choice of the best solution involves a careful evaluation of the advantages and disadvantages of each possible solution from the standpoint of performance, ease of manufacture, weight, reliability and many other factors. When a design is exceptionally cost conscious, it is reflected in assigning a large figure of merit to cost considerations in evaluating several possible structural solutions. While material A may be superior in a particular application material B may nevertheless be called out for the material of construction because of lower cost. A structural component can be made lighter and stronger by choosing solution A over solution B. Again, solution B may be incorporated into the design if manufacturing costs of the other are excessive. Numerous other examples can be cited in which one notes that both major and minor structural designs are involved. This subject is discussed in greater detail in another volume of this series.

4-7.3. Weight Considerations. Up to this point, only brief mention has been made of the importance of weight control in the design of a missile. The important advantages that are realized in a weight-conscious design are apparent when performance characteristics are studied. Each pound of weight saved is reflected either in an increase in the flight performance of the missile or, for the same weight, an increase in the payload is made possible. The missile structural designer is always weight-conscious notwithstanding the fact that the structural weight may be only of the order of 5 percent of the missile take-off gross weight.

Weight-consciousness is reflected in the structural design in a number of ways. In the selection of the material of construction, it means that the weight-strength characteristics must be given serious consideration. Unfortunately, no simple criterion exists for the weight-strength characteristic of a material. If Young's Modulus is chosen as a strength criterion, it is noted that most common metals of construction have approximately the same ratio of density to Young's Modulus. Molybdenum and tungsten, however, are notable exceptions. The ratio of density to the yield stress of a material is generally taken as the best indication of its weight-strength ratio. One of the principal reasons that a single simple weight-strength criterion is not decisive, lies in the fact that the true weight-strength property of a structural element is dependent on its configuration and the type of load that it is re-

quired to transmit. Further complication is introduced by the variation of any weight-strength criterion with temperature.

The efforts of structural designers to save weight also manifests itself in the selection of the most efficient structural configuration consistent with the physical properties of materials that are available. The reinforced monocoque construction is a noteworthy example. In weight-conscious designs, factors of safety are kept as low as possible and the most powerful methods of stress-analysis methods are used. Needless to say, the structural designer carries his weight-saving philosophy down to the most minute structural detail.

It is not to be implied, however, that missile structural designers consider weight-saving of paramount importance. Changes in one design feature invariably affect other characteristics of the missile system. Figures of merit are assigned as intelligently and rationally as possible, and compromises must be made. In making decisions on whether to change weight or fixing weight objectives, many factors must be considered by the designer. Some of these factors have already been mentioned, such as cost, effect on schedule, manufacturing ease, and effect on performance. There are other considerations that can be mentioned such as questions relating to the importance of the design function of the structural element, structural integrity, safety and reliability, effects on handling and launching equipment. Complete weight optimization, in short, is expensive, time consuming, and generally not warranted. One may consider having reached a point of diminishing returns in trying to refine say a 90 percent weight-optimized missile, in view of the excessive cost and time requirements to achieve the additional 10 percent weight optimization.

In the airplane design field, the wealth of weight data accumulated over the years has made it possible to obtain preliminary structural weight estimates by statistical and analytical methods. Unfortunately, the data available and applicable to missiles is relatively meager. This may be partially explained by the relative infancy and rapidly changing characteristic of the missile design field. In addition, ready access to weight data has been curtailed by security considerations. However, a coarse weight breakdown for the Redstone A missile, which has a range of 200-250 miles, is available. This information is presented in Table 4-1.

Depending upon the range, mission, and other parameters, the gross weight of missiles will vary widely. The portion of the total weight chargeable to payload, fuel, engine, structure and so forth will also vary for different missiles. Hence, no general conclusions should be drawn based on the weight data given for the Redstone.

TABLE 4-1.

WEIGHT BREAKDOWN FOR REDSTONE A MISSILE

	Approximate Weight (lbs)
Top Unit	12,296
Booster Unit	6,757
Engine	1,500
Payload	6,400
Total Weight	
Empty	19,053
Full	63,135
Length 820 inches	
Diameter 70 inches	

4-7.4. Reliability. The question of reliability, treated in greater detail in another volume of this series, may enter into and affect the choice of the material of construction, structural configuration, and cross-sectional area. A simplified example can be taken to show the effect that reliability considerations may have on either the selection of material or the cross-section area. Even when a structural material having well-established mechanical properties is employed in a design, it is necessary to establish the uniformity of the material properties by statistical testing of various batch and lots. Consider, on the other hand [illegible] use

of a material of inconsistent physical properties. The scatter of tests about the sample mean or median of a strength characteristic would dictate a lower design allowable, and hence a larger cross-sectional area for a given applied stress. In a preliminary design evaluation this may well lead to the structural designer to choose a material of more consistent physical properties.

4-7.5. Safety. The philosophy of design for safety is somewhat different in ballistic missiles than it is in the design of manned flight vehicles. In ballistic missile design the physical safety of flight personnel need not be considered. Thus the structural designer can keep safety factors at a minimum, especially for well-established design conditions and for secondary structures, when he can choose a structural material without the necessity of considering the potential loss of life. The possibility of structural failure near the launching platform, however, does involve physical safety of personnel, and cannot be overlooked.

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